

ARITHMETICAL ESSENTIALS

EIGHTH

GRADE



MAXIMUM COURSE

ARITHMETICAL ESSENTIALS

EIGHTH GRADE

Maximum Course

BY

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HELPS AND SUGGESTIONS TO TEACHERS

Provision for Pupils of Varying Ability

This book is intended to provide the largest possible opportunity for self instruction for children of the seventh and eighth grades. Sufficient material is provided for the superior pupils of the class. The teacher should be the judge of how much the slower children can accomplish.

Computing Skill

This text provides abundant practice in computation for children in the upper grades: first, through a series of timed practice exercises in each year; second, through the use of statistics in table form; third, through the application of short methods. See Chapter I, pp. 201-213; Chapter II, pp. 214-225; and Chapter XIII, pp. 361-402, for examples of each of these types.

The tests listed on pages 189 and 190 of the seventh grade text provide standardized material which teachers should use frequently during the eighth year to measure their classes. 100% accuracy with reasonable speed should be the aim in these computing exercises. Answers to examples are omitted from this book to emphasize the need of checking computations.

Certain 4 minute, 6 minute and 8 minute tests in the fundamental processes with integers are standardized against the corresponding Courtis Research Tests, Series B. This enables the teacher to use the Courtis Standards in these tests. The standards stated in all other exercises have been derived from enough cases to insure their validity.

Short Methods

It is not intended that children should memorize all the short forms in the chapter, **Saving Time in Computing**. The short

processes that seem most suitable for individual classes should be selected by each teacher for special emphasis. Using short methods should be encouraged whenever possible because it decreases the chances of error and saves time. Children who wish to develop special computing skill will find it valuable to do all the work in the chapter named.

Problems

The problem material of this book aims to give pupils an acquaintance with civic, social and business practices.

Three methods of showing the reasoning in a problem are suggested; the graphic form, the table form, and the equation form.

In the computing part of a problem pupils are encouraged to take the shortest road to the answer and to be certain that the answer is right. Pupils should decide for themselves whether or not their skill in computing demands the use of a pencil.

The Use of Graphs

A full discussion of graphs as instruments or tools to be used in making number relations obvious is presented early in the seventh year in order to give pupils ample time to acquire proficiency in a method, which has frequent application in ordinary reading and in everyday business. Pupils should be encouraged to collect local material to be used in constructing graphs, to graph results of their school activities, and to bring to class from their outside reading what they consider to be unusually good examples of graphs.

Square Root

In the extraction of square root the method of approximation by long division is employed in this text because this method is readily rationalized and easily remembered. This method requires the pupil to know division of integers and of decimals, and the meaning of a square root.

Metric System

It is intended to give a reading knowledge of the more common units of measure (meter, millimeter, gram, and liter) incidentally in connection with problem solving.

The more common metric tables are placed in the latter part of the book in parallel with the corresponding English tables as an aid to the teacher in case she wishes to teach metric units more intensively than is provided for in the text.

Organization

Organization of material is not only an essential element in the learning process, but it also is a necessary factor in one's ability to use what has been learned.

The material of this book is organized for the supervisor, the teacher, and the learner. The chapter titles indicate to the supervisor the order of development that is intended for the course. The chapter analysis as shown in the table of contents should guide the teacher in the presentation of the different topics. The development of the topic in the text is for the learner.

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ARITHMETICAL ESSENTIALS

BOOK SIX—EIGHTH YEAR

CHAPTER I

SHORT TESTS IN THE FUNDAMENTALS FOR SPEED AND ACCURACY*

Before finishing the 8th grade you should be able to do the number right indicated in each exercise in the time allotted if you are a standard pupil in that particular exercise. If you finish an exercise in less than the given time, you should spend the remainder in checking your work. It is more important to be accurate than to attempt more than the standard.

I

Addition—All right in one minute.

1. $\begin{array}{r} 2 \\ 4 \end{array}$ $\begin{array}{r} 3 \\ 8 \end{array}$ $\begin{array}{r} 9 \\ 5 \end{array}$ $\begin{array}{r} 4 \\ 2 \end{array}$ $\begin{array}{r} 6 \\ 6 \end{array}$ $\begin{array}{r} 8 \\ 0 \end{array}$ $\begin{array}{r} 4 \\ 3 \end{array}$ $\begin{array}{r} 6 \\ 9 \end{array}$ $\begin{array}{r} 4 \\ 5 \end{array}$ $\begin{array}{r} 3 \\ 2 \end{array}$ $\begin{array}{r} 2 \\ 7 \end{array}$ $\begin{array}{r} 9 \\ 9 \end{array}$ $\begin{array}{r} 7 \\ 6 \end{array}$ $\begin{array}{r} 7 \\ 7 \end{array}$ $\begin{array}{r} 2 \\ 5 \end{array}$

2. $\begin{array}{r} 4 \\ 7 \end{array}$ $\begin{array}{r} 8 \\ 9 \end{array}$ $\begin{array}{r} 4 \\ 6 \end{array}$ $\begin{array}{r} 7 \\ 9 \end{array}$ $\begin{array}{r} 3 \\ 5 \end{array}$ $\begin{array}{r} 8 \\ 3 \end{array}$ $\begin{array}{r} 2 \\ 9 \end{array}$ $\begin{array}{r} 5 \\ 8 \end{array}$ $\begin{array}{r} 6 \\ 5 \end{array}$ $\begin{array}{r} 7 \\ 2 \end{array}$ $\begin{array}{r} 9 \\ 0 \end{array}$ $\begin{array}{r} 9 \\ 6 \end{array}$ $\begin{array}{r} 3 \\ 4 \end{array}$ $\begin{array}{r} 9 \\ 3 \end{array}$ $\begin{array}{r} 7 \\ 8 \end{array}$

3. $\begin{array}{r} 9 \\ 1 \end{array}$ $\begin{array}{r} 5 \\ 2 \end{array}$ $\begin{array}{r} 2 \\ 8 \end{array}$ $\begin{array}{r} 4 \\ 9 \end{array}$ $\begin{array}{r} 8 \\ 6 \end{array}$ $\begin{array}{r} 4 \\ 4 \end{array}$ $\begin{array}{r} 5 \\ 7 \end{array}$ $\begin{array}{r} 6 \\ 3 \end{array}$ $\begin{array}{r} 9 \\ 8 \end{array}$ $\begin{array}{r} 4 \\ 8 \end{array}$ $\begin{array}{r} 5 \\ 9 \end{array}$ $\begin{array}{r} 5 \\ 5 \end{array}$ $\begin{array}{r} 8 \\ 2 \end{array}$ $\begin{array}{r} 3 \\ 3 \end{array}$ $\begin{array}{r} 7 \\ 4 \end{array}$

* NOTE.—In most cases the answers can be written on a slip of paper laid below the examples. Where examples need to be transferred, additional time should be allowed.

II

Subtraction—All right in one minute.

1.	1	4	5	6	3	2	7	3	4	5	4	9	7	6	9
	<u>1</u>	<u>1</u>	<u>4</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>4</u>	<u>5</u>	<u>5</u>	<u>6</u>	<u>2</u>
2.	5	7	6	7	6	6	3	5	8	7	4	7	9	8	8
	<u>5</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>6</u>	<u>3</u>	<u>7</u>	<u>1</u>	<u>2</u>	<u>8</u>
3.	2	8	9	8	9	9	8	6	9	8	9	9	5	8	7
	<u>2</u>	<u>3</u>	<u>9</u>	<u>7</u>	<u>8</u>	<u>3</u>	<u>6</u>	<u>3</u>	<u>7</u>	<u>4</u>	<u>6</u>	<u>4</u>	<u>3</u>	<u>5</u>	<u>4</u>

III

Subtraction—All right in one minute.

1.	18	10	16	12	17	15	12	15	13	14	16	10	11	10	10
	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>8</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>8</u>	<u>7</u>	<u>7</u>	<u>6</u>	<u>4</u>	<u>1</u>
2.	17	14	12	13	14	10	12	11	12	11	16	11	14	10	13
	<u>9</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>3</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>5</u>	<u>2</u>	<u>4</u>
3.	13	10	14	13	10	10	15	12	11	11	12	11	15	13	11
	<u>9</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>3</u>	<u>7</u>	<u>6</u>	<u>3</u>	<u>2</u>	<u>4</u>	<u>8</u>	<u>9</u>	<u>8</u>	<u>5</u>

IV

Multiplication—All right in one minute.

1.	3	4	9	0	5	4	3	2	7	4	9	6	5	4	7
	<u>2</u>	<u>7</u>	<u>8</u>	<u>2</u>	<u>6</u>	<u>1</u>	<u>6</u>	<u>9</u>	<u>6</u>	<u>0</u>	<u>5</u>	<u>9</u>	<u>2</u>	<u>8</u>	<u>0</u>
2.	6	2	9	0	7	9	2	7	0	8	9	3	9	2	4
	<u>5</u>	<u>7</u>	<u>6</u>	<u>5</u>	<u>4</u>	<u>7</u>	<u>8</u>	<u>7</u>	<u>6</u>	<u>3</u>	<u>2</u>	<u>8</u>	<u>9</u>	<u>0</u>	<u>3</u>
3.	4	8	0	4	6	8	0	9	3	6	3	6	7	5	4
	<u>4</u>	<u>9</u>	<u>3</u>	<u>5</u>	<u>2</u>	<u>8</u>	<u>7</u>	<u>3</u>	<u>4</u>	<u>8</u>	<u>9</u>	<u>3</u>	<u>9</u>	<u>5</u>	<u>6</u>

V

Division—All right in one minute.

1. $3\overline{)9}$	$4\overline{)32}$	$6\overline{)36}$	$2\overline{)0}$	$7\overline{)28}$	$9\overline{)9}$	$3\overline{)21}$	$6\overline{)48}$	$1\overline{)1}$
2. $5\overline{)10}$	$4\overline{)24}$	$7\overline{)63}$	$6\overline{)0}$	$8\overline{)32}$	$1\overline{)8}$	$5\overline{)30}$	$8\overline{)72}$	$1\overline{)0}$
3. $9\overline{)36}$	$1\overline{)7}$	$2\overline{)10}$	$7\overline{)42}$	$1\overline{)1}$	$6\overline{)18}$	$4\overline{)20}$	$7\overline{)49}$	$1\overline{)3}$
4. $2\overline{)8}$	$6\overline{)6}$	$3\overline{)27}$	$8\overline{)64}$	$1\overline{)2}$	$5\overline{)0}$	$3\overline{)24}$	$9\overline{)63}$	$8\overline{)24}$
5. $7\overline{)7}$	$2\overline{)18}$	$6\overline{)42}$	$3\overline{)0}$	$7\overline{)21}$	$4\overline{)4}$	$3\overline{)15}$	$9\overline{)81}$	$7\overline{)0}$

VI

Addition—5 right in one minute.

56	22	96	82	56	14	41	39
27	78	82	85	93	40	41	39
28	38	30	15	64	28	51	12
15	45	79	36	83	55	38	59
<u>45</u>	<u>11</u>	<u>63</u>	<u>80</u>	<u>78</u>	<u>46</u>	<u>44</u>	<u>76</u>

VII

Subtraction—10 right in one minute.

971	821	934	408	725	561	733
<u>363</u>	<u>130</u>	<u>738</u>	<u>265</u>	<u>482</u>	<u>172</u>	<u>507</u>
708	616	854	365	905	496	1001
<u>578</u>	<u>259</u>	<u>286</u>	<u>178</u>	<u>286</u>	<u>327</u>	<u>842</u>

VIII

Multiplication—All right in one minute.

2534	3075	7689	6879	3245	2354	3524
<u>5</u>	<u>2</u>	<u>5</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>7</u>

IX

Division—4 right in one minute.

$$\begin{array}{r} 4 \overline{)38968} \end{array} \quad \begin{array}{r} 6 \overline{)21279} \end{array} \quad \begin{array}{r} 7 \overline{)39872} \end{array} \quad \begin{array}{r} 8 \overline{)66720} \end{array} \quad \begin{array}{r} 9 \overline{)71289} \end{array}$$

X

Addition—6 right in 3 minutes.

8685	7521	2846	8253	7296	3580	4765
4944	2754	6389	7594	6802	6295	3698
5636	4856	3528	6031	3690	3782	8787
3678	7039	3893	7426	3285	9061	7669
<u>3869</u>	<u>9483</u>	<u>4678</u>	<u>7426</u>	<u>9735</u>	<u>3648</u>	<u>8327</u>

XI

Multiplication—5 right in 3 minutes.

43957	52468	73098	87692	30296	65382
<u>73</u>	<u>46</u>	<u>29</u>	<u>37</u>	<u>31</u>	<u>48</u>

XII

Long Division—3 right in 3 minutes.

$$\begin{array}{r} 32 \overline{)793664} \end{array} \quad \begin{array}{r} 46 \overline{)284602} \end{array} \quad \begin{array}{r} 97 \overline{)102626} \end{array} \quad \begin{array}{r} 88 \overline{)671792} \end{array}$$

XIII

Addition—10 right in 3 minutes.

$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{11}{12}$	$\frac{11}{24}$	$\frac{1}{32}$	$\frac{7}{8}$	$\frac{5}{16}$
$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{5}{8}$	$\frac{7}{12}$	$\frac{5}{8}$
$\frac{5}{8}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{7}{12}$	$\frac{1}{3}$	$\frac{13}{16}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{5}{24}$	$\frac{3}{16}$	$\frac{7}{24}$	$\frac{1}{4}$

XIV

Addition—12 right in eight minutes.

234	783	134	215	245	864	319	932
979	420	979	787	306	977	985	749
567	828	305	478	498	590	758	288
478	539	887	196	589	745	264	539
754	354	260	509	250	839	437	672
992	196	456	924	615	986	801	416
366	217	392	840	824	692	126	284
438	604	910	382	707	548	598	219
<u>329</u>	<u>975</u>	<u>129</u>	<u>623</u>	<u>191</u>	<u>704</u>	<u>677</u>	<u>704</u>
577	205	586	170	208	217	586	359
194	110	324	676	984	173	324	235
538	324	247	505	352	538	247	714
636	638	174	237	745	625	170	129
860	411	958	458	497	209	958	647
575	539	435	722	176	994	435	280
751	747	913	384	569	380	913	486
402	156	692	899	623	456	692	592
<u>389</u>	<u>200</u>	<u>709</u>	<u>968</u>	<u>838</u>	<u>661</u>	<u>709</u>	<u>678</u>

XV

This exercise contains 90 subtraction facts—All right in 5 minutes.

80314927	8760051	4003347	82364354
<u>46349441</u>	<u>2273528</u>	<u>2712617</u>	<u>35845858</u>
8590323	42754101	2558496	93832917
<u>5962665</u>	<u>32851519</u>	<u>1412743</u>	<u>32105144</u>
7598719	76697666	9562145	80802121
<u>4887916</u>	<u>22416569</u>	<u>9208247</u>	<u>65887712</u>

XVI

Multiplication—11 right in six minutes.

2143	4079	6785	9809	9786	6739	4921	9524
<u>89</u>	<u>57</u>	<u>96</u>	<u>42</u>	<u>45</u>	<u>19</u>	<u>73</u>	<u>58</u>
6785	7568	3478	3484	3792	3098	8742	5624
<u>24</u>	<u>83</u>	<u>92</u>	<u>208</u>	<u>57</u>	<u>67</u>	<u>360</u>	<u>18</u>

XVII

These examples contain 90 multiplication facts—All right in six minutes.

45239	53429	60871	39245	87160	71608
<u>278</u>	<u>951</u>	<u>273</u>	<u>436</u>	<u>916</u>	<u>845</u>

XVIII

Division—11 right in eight minutes.

28)8204	65)45825	58)51736	94)38634	47)39997
36)23076	74)48100	25)12075	93)69006	76)29184
49)34986	52)14196	34)31960	85)28685	56)45304

XIX

Subtraction—All right in three minutes.

$2\frac{7}{8}$	$6\frac{15}{16}$	$2\frac{11}{12}$	$9\frac{5}{8}$	$6\frac{1}{2}$	$16\frac{1}{2}$	24	$18\frac{1}{4}$
<u>$1\frac{3}{4}$</u>	<u>$3\frac{7}{8}$</u>	<u>$1\frac{2}{3}$</u>	<u>$6\frac{1}{2}$</u>	<u>$2\frac{3}{4}$</u>	<u>$12\frac{3}{4}$</u>	<u>$15\frac{3}{8}$</u>	<u>$16\frac{1}{2}$</u>
$3\frac{1}{3}$	100	100 %	200	200	50 %	75 %	$12\frac{1}{2}$
<u>$2\frac{1}{4}$</u>	<u>$66\frac{2}{3}$</u>	<u>$62\frac{1}{2}\%$</u>	<u>$137\frac{1}{2}$</u>	<u>$187\frac{1}{2}$</u>	<u>$33\frac{1}{3}\%$</u>	<u>$37\frac{1}{2}\%$</u>	<u>$7\frac{15}{16}$</u>
$18\frac{2}{3}$	50 ¢	25 %	$\frac{1}{4}$	$87\frac{1}{2}\%$	$\frac{7}{8}$	$16\frac{2}{3}\%$	$\frac{1}{6}$
<u>$5\frac{3}{4}$</u>	<u>$12\frac{1}{2}\text{¢}$</u>	<u>$8\frac{1}{3}\%$</u>	<u>$\frac{1}{12}$</u>	<u>$16\frac{2}{3}\%$</u>	<u>$\frac{1}{6}$</u>	<u>$6\frac{1}{4}\%$</u>	<u>$\frac{1}{16}$</u>

XX

Use of Signs—All right in five minutes.

- | | |
|--------------------------------------|---|
| 1. $(8 \times 4) + (3 \times 5) = ?$ | 11. $36 - (18 \div 6) = ?$ |
| 2. $5 + (8 \times 5) = ?$ | 12. $(3 \times 4) + (8 \times 5) - (12 \div 6) = ?$ |
| 3. $(6 \div 2) + (3 \times 10) = ?$ | 13. $(5 \times 6) - (2 \div 4) + (7 \times 2) = ?$ |
| 4. $(15 \div 5) + (21 \div 7) = ?$ | 14. $(6 \times \frac{1}{2}) + (8 \div 4 \times 7) = ?$ |
| 5. $(24 \div 6) - (34 \div 17) = ?$ | 15. $(7 \times 8 \div 4) - (6 \times 2) - (8 \div 8) = ?$ |
| 6. $25 \times 2 \div 10 = ?$ | 16. $3 \times 52 + 7 = ?$ |
| 7. $18 \div 6 \times 5 = ?$ | 17. $(18 \div 6) + (7 \times 4) - (24 \div 8 \times 4) = ?$ |
| 8. $27 \div 3 \times 4 - 5 = ?$ | 18. $(8 + 4) \times (16 - 12) = ?$ |
| 9. $25 - 10 + 5 = ?$ | 19. $(7 - 3) \times (8 + 2) - (6 \div 2) = ?$ |
| 10. $18 + (4 \div 2) = ?$ | 20. $(14 \div 2) + (4 - 3) + (6 \times 8) = ?$ |

XXI

There is an easy way to add the following by grouping certain numbers. Find it. Then add.

All right in six minutes.

84%	$2\frac{1}{5}$	$64\frac{3}{8}$	$16\frac{2}{3}$	175	$37\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$	15
16%	$3\frac{1}{2}$	$25\frac{1}{4}$	$8\frac{1}{12}$	225	$62\frac{1}{2}$	$\frac{5}{8}$	$\frac{7}{12}$	34
65%	$4\frac{3}{4}$	$19\frac{1}{2}$	$15\frac{1}{4}$	845	50	$\frac{1}{6}$	$\frac{1}{4}$	25
27%	$5\frac{4}{5}$	$65\frac{7}{8}$	$22\frac{1}{2}$	155	$33\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	66
73%	$8\frac{1}{4}$	19	$34\frac{1}{6}$	450	40	$\frac{3}{8}$	$3\frac{1}{3}$	85
35%	$6\frac{1}{2}$	$24\frac{1}{8}$	30	350	$66\frac{2}{3}$	$\frac{1}{4}$	$4\frac{1}{2}$	50

XXII

These division examples require all the multiplication facts if the right answers are obtained.

All right in ten minutes.

71608) <u>60508760</u>	87160) <u>79838560</u>	39245) <u>17110820</u>
45239) <u>12576442</u>	53429) <u>50810979</u>	60871) <u>16617783</u>

XXIII

A Fraction Test

Read across the page; write answers in lowest terms.

You should have 100 right in 25 minutes.

	a	b	c	d	e
1.	$\frac{1}{2} + \frac{1}{4}$	$\frac{1}{4} + \frac{1}{8}$	$\frac{1}{3} + \frac{1}{6}$	$\frac{1}{5} + \frac{1}{10}$	$\frac{1}{3} + \frac{1}{4}$
2.	$\frac{3}{4} + \frac{1}{8}$	$\frac{2}{3} + \frac{1}{6}$	$\frac{2}{5} + \frac{1}{10}$	$\frac{2}{3} + \frac{1}{9}$	$\frac{1}{3} + \frac{2}{9}$
3.	$\frac{1}{2} - \frac{1}{4}$	$\frac{1}{4} - \frac{1}{8}$	$\frac{3}{4} - \frac{5}{8}$	$\frac{1}{5} - \frac{1}{10}$	$\frac{2}{5} - \frac{3}{10}$
4.	$\frac{1}{3} - \frac{1}{6}$	$\frac{1}{3} - \frac{1}{9}$	$\frac{2}{3} - \frac{1}{6}$	$\frac{2}{3} - \frac{4}{9}$	$\frac{2}{3} - \frac{1}{2}$
5.	$\frac{1}{2} \times 3$	$\frac{1}{3} \times 4$	$\frac{1}{4} \times 6$	$\frac{1}{5} \times 6$	$\frac{1}{6} \times 4$
6.	$\frac{2}{3} \times 4$	$\frac{3}{4} \times 4$	$\frac{5}{6} \times 4$	$\frac{2}{5} \times 4$	$\frac{3}{5} \times 5$
7.	$\frac{3}{5} \div 3$	$\frac{6}{12} \div 2$	$\frac{8}{9} \div 4$	$\frac{12}{16} \div 3$	$\frac{18}{25} \div 6$
8.	$\frac{1}{2} \div \frac{1}{2}$	$\frac{1}{2} \div \frac{1}{3}$	$\frac{1}{3} \div \frac{1}{2}$	$\frac{2}{3} \div \frac{3}{4}$	$\frac{3}{4} \div \frac{2}{3}$
9.	$\frac{2}{3} \div 2$	$\frac{1}{3} \div 2$	$\frac{3}{5} \div 4$	$\frac{5}{3} \div 3$	$\frac{3}{4} \div 5$
10.	$\frac{2}{5} + \frac{3}{10}$	$\frac{7}{10} + \frac{2}{5}$	$\frac{7}{10} \times 5$	$\frac{18}{9} \div 9$	$\frac{5}{8} \times \frac{1}{2}$
11.	$\frac{1}{2} - \frac{3}{8}$	$\frac{4}{5} \times 4$	$\frac{21}{2} \div 3$	$\frac{1}{4} + \frac{1}{3}$	$\frac{1}{3} - \frac{1}{4}$

a	b	c	d	e
12. $2\frac{1}{5} \times 4$	$15\frac{5}{8} \div 5$	$\frac{1}{2} + 2\frac{1}{4}$	$1\frac{1}{2} - \frac{3}{4}$	$\frac{2}{3} \times \frac{9}{16}$
13. $3\frac{3}{4} \div 3$	$8\frac{4}{7} \div 4$	$10\frac{5}{8} \div 5$	$12\frac{8}{9} \div 4$	$18\frac{2}{3} \div 6$
14. $\frac{21}{4} \div 7$	$\frac{6}{7} \div 6$	$\frac{12}{16} \div 4$	$\frac{24}{25} \div 8$	$\frac{27}{28} \div 9$
15. $\frac{3}{5} - \frac{1}{10}$	$2\frac{1}{2} - 1\frac{1}{4}$	$3\frac{1}{3} + 2\frac{1}{6}$	$\frac{4}{9} \times \frac{6}{12}$	$\frac{1}{10} \div \frac{3}{5}$
16. $\frac{1}{8} + \frac{1}{2}$	$\frac{1}{6} + \frac{1}{2}$	$\frac{1}{10} + \frac{1}{2}$	$\frac{5}{6} + \frac{2}{3}$	$\frac{3}{8} + \frac{1}{2}$
17. $\frac{5}{6} - \frac{1}{3}$	$\frac{5}{8} - \frac{1}{3}$	$\frac{5}{6} - \frac{2}{3}$	$\frac{7}{9} - \frac{2}{3}$	$\frac{5}{6} - \frac{1}{2}$
18. $\frac{4}{5} \times 3$	$\frac{2}{5} \times 6$	$\frac{1}{7} \times 9$	$\frac{2}{7} \times 5$	$\frac{3}{7} \times 3$
19. $\frac{1}{2} \times \frac{2}{3}$	$\frac{3}{4} \times \frac{2}{3}$	$\frac{5}{8} \times \frac{2}{5}$	$\frac{3}{8} \times \frac{4}{9}$	$\frac{3}{4} \times \frac{5}{12}$
20. $\frac{1}{2} - \frac{3}{10}$	$\frac{1}{2} + \frac{3}{4}$	$\frac{1}{2} \div \frac{5}{6}$	$\frac{1}{2} + \frac{5}{8}$	$\frac{1}{2} + \frac{7}{10}$
21. $\frac{3}{4} + \frac{1}{6}$	$\frac{7}{8} - \frac{3}{4}$	$8 \times \frac{3}{16}$	$\frac{8}{9} \times 3$	$5 \div 2\frac{1}{2}$
22. $2\frac{1}{2} \times 8$	$\frac{3}{4} + \frac{5}{16}$	$\frac{9}{10} \div \frac{2}{5}$	$\frac{3}{4} \div 4$	$7 \div 1\frac{1}{2}$
23. $6 - 1\frac{1}{2}$	$\frac{5}{8} - \frac{13}{24}$	$\frac{5}{6} + 1\frac{3}{4}$	$2\frac{1}{2} + 2\frac{3}{4}$	$2 \times 5\frac{3}{4}$

XXIV

Write on another sheet each of the following quotients with the decimal point properly placed.

Do all right in three minutes.

- | | | | |
|------------------------------|----------------------------|----------------------------|------------------------------|
| (1) $12 \overline{)12}$ | (2) $.1 \overline{)10}$ | (3) $1.2 \overline{)2.4}$ | (4) $.24 \overline{)1.20}$ |
| (5) $.25 \overline{)50}$ | (6) $.7 \overline{)3.5}$ | (7) $.7 \overline{).56}$ | (8) $.25 \overline{).25}$ |
| (9) $.8 \overline{)8}$ | (10) $.9 \overline{)0.36}$ | (11) $.3 \overline{)3}$ | (12) $.021 \overline{).84}$ |
| (13) $81 \overline{)3.24}$ | (14) $.3 \overline{).30}$ | (15) $.005 \overline{)25}$ | (16) $.25 \overline{)800}$ |
| (17) $8.5 \overline{)722.5}$ | (18) $9 \overline{).036}$ | (19) $.4 \overline{)3.28}$ | (20) $1.9 \overline{).0399}$ |

XXV

In each of the following, write the quotient without rewriting the example if possible.

Do 40 right in ten minutes.

- | a | b | c |
|-----------------------|---------------------|--------------------------|
| 1. $12 \div 12 = ?$ | $.12 \div 12 = ?$ | $.01 \div .001 = ?$ |
| 2. $10 \div .1 = ?$ | $75 \div 25 = ?$ | $75 \div .25 = ?$ |
| 3. $84 \div 840 = ?$ | $84 \div 8.4 = ?$ | $.84 \div 21 = ?$ |
| 4. $3.24 \div 81 = ?$ | $.84 \div .021 = ?$ | $3 \div \frac{1}{3} = ?$ |
| 5. $3 \div .3 = ?$ | $30 \div .03 = ?$ | $.30 \div .3 = ?$ |
| 6. $2.4 \div 1.2 = ?$ | $1.2 \div 2.4 = ?$ | $.12 \div .24 = ?$ |
| 7. $.12 \div 2.4 = ?$ | $.036 \div 9 = ?$ | $0.36 \div .9 = ?$ |

a	b	c
8. $36 \div 90 = ?$	$15 \div 150 = ?$	$27 \div 2.7 = ?$
9. $4 \div 40 = ?$	$.4 \div .04 = ?$	$40 \div 4 = ?$
10. $100 \div .25 = ?$	$800 \div .25 = ?$	$250 \div 50 = ?$
11. $50 \div 250 = ?$	$16 \div 64 = ?$	$3.5 \div .7 = ?$
12. $45 \div .9 = ?$	$4.5 \div .9 = ?$	$.02 \div .1 = ?$
13. $56 \div .7 = ?$	$.7 \div .14 = ?$	$2.5 \div 25 = ?$
14. $25 \div .25 = ?$	$2.5 \div .25 = ?$	$.25 \div .25 = ?$
15. $80 \div .8 = ?$	$8.0 \div .8 = ?$	$8 \div 80 = ?$

XXVI

Review the short methods in multiplication, then solve the following examples in the most economical way. Time yourself.

a	b	c
1. $76 \times 25 = ?$	$99 \times 33\frac{1}{3} = ?$	$\$11.25 \times 40 = ?$
2. $800 \times 62 = ?$	$75 \times 75 = ?$	$7\frac{1}{2} \times 7\frac{1}{2} = ?$
3. $1200 \times 2\frac{3}{4} = ?$	$\$18.75 \times 20 = ?$	$16\frac{2}{3} \times 240 = ?$
4. $16\frac{2}{3}\% \times 18 = ?$	$16\frac{2}{3} \times 48 = ?$	$12\frac{1}{2} \times 64 = ?$
5. $12\frac{1}{2}\% \times 84 = ?$	$12 \times 66\frac{2}{3} = ?$	$65 \times 19 = ?$
6. $49 \times 51 = ?$	$99 \times 7 = ?$	$.02\frac{1}{2} \times 16 = ?$
7. $.03\frac{1}{3} \times 90 = ?$	$.96\frac{2}{3} \times 300 = ?$	$101\% \times 70 = ?$
8. $125\% \times 16 = ?$	$11 \times \$120 = ?$	$25 \times \$36 = ?$
9. $\$55 \times 55 = ?$	$95 \times \$95 = ?$	$24.75 \times 4 = ?$
10. $25\% \times \$40.40 = ?$	$99 \times 64 = ?$	$39 \times 61 = ?$
11. $1.8 \times \$600 = ?$	$.19 \times 300 = ?$	$2\frac{1}{2} \times 2\frac{1}{2} = ?$
12. $12 \times 144 = ?$	$35 \times 35 = ?$	$125 \times 6.4 = ?$
13. $133\frac{1}{3}\% \times 900 = ?$	$125\% \times 12 = ?$	$200\% \times \$25.75 = ?$
14. $.00\frac{1}{2} \times \$1000 = ?$	$\frac{3}{4}\% \times 1600 \text{ ft.} = ?$	$.05\% \times \$10000 = ?$
15. $175\% \times 44 = ?$	$15 \times 150 = ?$	$115 \times 115 = ?$

XXVII

Name the missing numbers.

The correct answer for No. 1 is "3 is $\frac{1}{2}$ of 6, or $\frac{1}{4}$ of 12, or 6% of 50, or $2 \times 1\frac{1}{2}$."

1. 3 is $\frac{1}{2}$ of __, or $\frac{1}{4}$ of __, or 6% of __, or $2 \times$ __
2. 10 is $\frac{2}{5}$ of __, or .1 of __, or $5 \times$ __, or $\frac{1}{6}$ of __
3. 50 is $\frac{2}{3}$ of __, or $1\frac{2}{3} \times$ __, or $2\frac{1}{2} \times$ __, or 20% of __
4. 24 is $\frac{4}{3}$ of __, or 75% of __, or 40% of __, or $2\frac{2}{3} \times$ __
5. $\frac{1}{4}$ is $\frac{1}{2}$ of __, or $\frac{1}{3}$ of __, or $3 \times$ __, or 50% of __
6. 27 is $\frac{3}{4}$ of __, or $33\frac{1}{3}\%$ of __, or $2\frac{1}{4} \times$ __, or $1\frac{1}{2} \times$ __
7. $\frac{3}{4}$ is $\frac{1}{3}$ of __, or $3 \times$ __, or 100% of __, or $4 \times$ __
8. 18 is $\frac{2}{3}$ of __, or $\frac{3}{4}$ of __, or $12\frac{1}{2}\%$ of __, or .6 of __
9. .8 is $\frac{4}{5}$ of __, or $2 \times$ __, or 50% of __, or $16\frac{2}{3}\%$ of __
10. 25 is 20% of __, or 25% of __, or 4% of __, or $2\frac{1}{2} \times$ __
11. 14 is $3\frac{1}{2} \times$ __, or $\frac{2}{3}$ of __, or $\frac{7}{8}$ of __, or 25% of __
12. 90 is $.9 \times$ __, or 10% of __, or $2\frac{1}{4} \times$ __, or $\frac{2}{3}$ of __

XXVIII

Find the average and the median.

1. $2\frac{3}{4}$, $3\frac{1}{2}$, $6\frac{1}{4}$, 4, $2\frac{1}{2}$
2. 8.5, 6.3, 7.2, 9.4, 6.1
3. .84, .75, .62, .56, .80
4. 80%, 90%, 85%, 95%, 88%
5. 60¢, 48¢, 55¢, 60¢, 50¢
6. 48.3 yd., 52.8 yd., 50.7 yd.
7. 8 yr. 6 mo., 9 yr. 3 mo., 10 yr. 2 mo.
8. 5 ft. 4 in., 5 ft. 8 in., 5 ft. 1 in.
9. $16\frac{1}{2}$ lb., $18\frac{3}{4}$ lb., $19\frac{1}{2}$ lb., $15\frac{1}{4}$ lb., 17 lb.
10. \$125, \$140, \$122, \$132, \$135

XXIX

Find the missing number.

- | | |
|--|--|
| 1. $\$3 = \underline{\hspace{1cm}} 25\text{¢}$ | 9. $\$2 = \underline{\hspace{1cm}} \times 8\frac{1}{3}\text{¢}$ |
| 2. $\$8 = 12\frac{1}{2}\text{¢} \times \underline{\hspace{1cm}}$ | 10. $\$1.50 = \underline{\hspace{1cm}} \times 12\frac{1}{2}\text{¢}$ |
| 3. $4 \times 37\frac{1}{2}\text{¢} = \underline{\hspace{1cm}}$ | 11. $\$6.25 = 25\text{¢} \times \underline{\hspace{1cm}}$ |
| 4. $\$12 = \underline{\hspace{1cm}} 50\text{¢}$ | 12. $15 \times \$2.50 = \underline{\hspace{1cm}}$ |
| 5. $\$2.25 = \underline{\hspace{1cm}} \times 25\text{¢}$ | 13. $16 \times \$2.25 = \underline{\hspace{1cm}}$ |
| 6. $\$22.50 = \$2.25 \times \underline{\hspace{1cm}}$ | 14. $24 \times 37\frac{1}{2}\text{¢} = \underline{\hspace{1cm}}$ |
| 7. $\$30 = \underline{\hspace{1cm}} \times \$3.75$ | 15. $32 \times 62\frac{1}{2}\text{¢} = \underline{\hspace{1cm}}$ |
| 8. $\$50 = \underline{\hspace{1cm}} \times \$2.50$ | 16. $\$60 = \underline{\hspace{1cm}} \times 75\text{¢}$ |

XXX

Are you 100% efficient in computation?

This test is given to those seeking employment in a certain department store in a large city. The passing mark is 100. The time must not exceed 10 minutes.

Try to solve all right in less than 10 minutes.

- | | | |
|---|--|--|
| 1. $\begin{array}{r} 98773 \\ 37521 \\ 46378 \\ 73216 \\ 53764 \\ +21976 \\ \hline \end{array}$ | 3. $\begin{array}{r} 75412 \\ \times 8 \\ \hline \end{array}$ | 6. Find the cost of each of the following: |
| Ans. $\begin{array}{r} 98773 \\ 37521 \\ 46378 \\ 73216 \\ 53764 \\ +21976 \\ \hline \end{array}$ | Ans. $\begin{array}{r} 75412 \\ \times 8 \\ \hline \end{array}$ | (1) 9 yd. gingham@ $12\frac{1}{2}\text{¢}$ Ans. |
| 2. $\begin{array}{r} 74621 \\ -43130 \\ \hline \end{array}$ | 4. $\begin{array}{r} 98744 \\ \times 78 \\ \hline \end{array}$ | (2) $12\frac{1}{2}$ yd. muslin@ 22¢ Ans. |
| Ans. $\begin{array}{r} 74621 \\ -43130 \\ \hline \end{array}$ | Ans. $\begin{array}{r} 98744 \\ \times 78 \\ \hline \end{array}$ | (3) 4 pr. lace curtains
@ $\$3.75$ pr. Ans. |
| | 5. $\begin{array}{r} 9)13437659 \\ \hline \end{array}$ | Less 10% discount Ans. |
| | Ans. $\begin{array}{r} 9)13437659 \\ \hline \end{array}$ | (4) $15\frac{1}{4}$ yd. lace@ $12\frac{1}{2}\text{¢}$ yd.
Less 25% discount Ans. |

CHAPTER II

SPEED AND ACCURACY IN COMPUTATION THROUGH SHORT METHODS

A pupil whose accuracy in computation is not better than 90%, wrong once in ten, should not hope to hold any position requiring much computing. Business men will not employ careless computers for positions of responsibility.

Short methods in operations are not only a time saving device, but they are also a means of securing greater accuracy.

A pupil who learns to get a required result by two operations which formerly required four has reduced through the short method his chances at error by at least 50%.

In Addition

I. By Group Adding.

1. Grouping the addition facts.

356 In column addition you should use in the manner here shown
482 the addition facts already learned.
769 Adding the 1st column, think 17, 27, 35. Write 5.
851 Adding the 2nd column, think 17, 28, 41. Write 1.
678 Adding the 3rd column, think 14, 29, 36. Write 36.
479 Some pupils easily group three figures if their sum does not exceed 12.
3615

1.	2.	3.	4.	5.	6.	7.
(7627)	(4381)	(4578)	4258	3264	4639	7824
(4432)	(6297)	(6536)	6479	7946	3066	4636
(3396)	(3811)	(2568)	3624	8309	4947	3899
(4623)	(7292)	(4479)	2457	4093	3073	6473
(9287)	(4623)	(2529)	2413	6377	6048	3868
(4630)	(3269)	(7342)	9850	4708	4724	7487
<u>3841</u>	<u>7426</u>	<u>3456</u>	<u>4620</u>	<u>3824</u>	<u>3966</u>	<u>4966</u>

2. Grouping by 100's.

In short two-figure columns it is often convenient to group the numbers which make 100. Try these examples in this way. Time yourself to-day. Check. Try again to-morrow.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
84	39	52	21	45	15	23	14	56	49
16	75	48	29	55	85	25	26	34	21
53	25	33	71	89	96	41	60	18	30
47	27	67	38	74	4	59	95	92	89
<u>35</u>	<u>73</u>	<u>19</u>	<u>62</u>	<u>26</u>	<u>72</u>	<u>75</u>	<u>5</u>	<u>73</u>	<u>22</u>

In short examples sometimes it is possible to add three columns at a time by grouping those numbers which make 100's or 150's. Try these examples in this way.

1.	2.	3.	4.	5.	6.	7.	8.	9.
125	258	364	562	822	875	325	283	587
375	142	189	388	278	325	875	217	113
250	567	211	420	327	264	395	642	256
<u>150</u>	<u>433</u>	<u>236</u>	<u>230</u>	<u>173</u>	<u>336</u>	<u>105</u>	<u>358</u>	<u>344</u>

II. By Line Adding.

1. Adding two numbers without pencil.

Add to the first number the largest decade of the second.

Then to this sum add the units of the second.

$45 + 38 = ?$ Think 45, 75, 83.

a	b	c
1. $29 + 18 = ?$	$58 + 34 = ?$	$65 + 28 = ?$
2. $72 + 38 = ?$	$37 + 65 = ?$	$85 + 29 = ?$
3. $62 + 49 = ?$	$39 + 59 = ?$	$47 + 67 = ?$
4. $53 + 83 = ?$	$76 + 85 = ?$	$94 + 56 = ?$
5. $87 + 56 = ?$	$79 + 97 = ?$	$36 + 83 = ?$
6. $82 + 92 = ?$	$48 + 72 = ?$	$57 + 77 = ?$

1. Subtract each of the following from 100.

Read the answers.

	a	b	c	d	e	f	g	h
1.	24	28	37	17	84	72	15	36
2.	56	33	48	71	27	23	32	61
3.	44	14	9	89	79	39	41	82
4.	63	55	85	77	65	75	13	87

2. Read the answers for these examples by thinking in this way, $84 - 26 = ?$ Think 20 from 84, 6 from 64. Say 58.

	a	b	c
1.	$96 - 32 = ?$	$48 - 29 = ?$	$62 - 23 = ?$
2.	$42 - 28 = ?$	$100 - 63 = ?$	$102 - 57 = ?$
3.	$145 - 76 = ?$	$114 - 68 = ?$	$121 - 96 = ?$
4.	$152 - 85 = ?$	$136 - 89 = ?$	$105 - 68 = ?$

3. Find the gross profit of each department, the net profit of each department, and the total profits. Prove the totals.

Net profit is the gross profit less the expense.

Summary for November, 1923, by Departments

Department	Sales Total	Cost of Goods Sold	Gross Profit	Expense	Net Profit
Furniture.....	\$4729.60	\$2886.75	?	\$1189.38	?
Dry Goods.....	6378.75	5635.97	?	935.07	?
Groceries.....	3811.17	2475.80	?	764.24	?
Clothing.....	2675.92	1709.78	?	538.54	?
Furnishings.....	3044.69	1974.37	?	679.88	?
Carpets.....	6879.54	4311.70	?	1014.34	?
Totals.....	?	?	??	?	??

In Multiplication

1. Aliquot parts of 100.

These are the important aliquot parts of 100. Save time in multiplying by knowing and using them.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{1}{16}$
50	$33\frac{1}{3}$	25	20	$16\frac{2}{3}$	$12\frac{1}{2}$	10	$8\frac{1}{3}$	$6\frac{1}{4}$

When articles are purchased at 25¢ each, how does the number of dollars in the cost compare with the number of articles?

Answer the same question for $12\frac{1}{2}$ ¢, for 20¢, for $33\frac{1}{3}$ ¢, for $16\frac{2}{3}$ ¢, for $8\frac{1}{3}$ ¢.

Using aliquot parts, read the answers to these examples. If the fraction in your answer is $\frac{1}{2}$ ¢ or more, call it 1¢.

a		b		c	
1.	32 lb. @ 25¢	18 ft. @ $33\frac{1}{3}$ ¢	34 ft. @ $12\frac{1}{2}$ ¢		
2.	37 lb. @ 50¢	49 lb. @ 25¢	84 ft. @ $12\frac{1}{2}$ ¢		
3.	31 yd. @ 25¢	48 yd. @ $6\frac{1}{4}$ ¢	72 yd. @ $6\frac{1}{4}$ ¢		
4.	375 yd. @ $33\frac{1}{3}$ ¢	336 gal. @ $8\frac{1}{3}$ ¢	639 yd. @ $33\frac{1}{3}$ ¢		
5.	576 ft. @ $16\frac{2}{3}$ ¢	756 ft. @ $8\frac{1}{3}$ ¢	384 gal. @ $33\frac{1}{3}$ ¢		
6.	56 qt. @ $8\frac{1}{3}$ ¢	56 doz. @ $16\frac{2}{3}$ ¢	38 yd. @ $8\frac{1}{3}$ ¢		

Interchanging Price and Quantity

If a problem be stated, "Find the cost of 25 yd. at 18¢," it may be solved as though it read, "Find the cost of 18 yd. at 25¢," because the answer is the same in both problems. Changing the order of factors in multiplication does not change the product.

Find the cost of each of the following by interchanging price and quantity.

a	b	c
1. 25 yd. @ 26¢	12½ yd. @ 14¢	6¼ yd. @ 68¢
2. 25 gal. @ 83¢	25 lb. @ 9¢	8⅓ yd. @ \$2.40
3. 16⅔ yd. @ 54¢	33⅓ yd. @ 24¢	12½ yd. @ 45¢
4. 25 doz. @ 45¢	8 doz. @ 48¢	12½ doz. @ 56¢

2. When the parts of one of the factors have an easy multiple relation to each other.

(a) $246 \times 846 = ?$	(b) $348 \times 945 = ?$
846 By making 246 the multiplier, the partial products	348 In (b) multiply
246 tiplier, the partial products	945 first by 9. Multi-
5076 reduce to two. $24 = 4 \times 6$.	3132 ply this product
20304 Multiply by 6. Then this	15660 by 5. Why?
208116 product by 4. Why?	328860

Solve by the long method examples (a) and (b) above, then count how many steps in multiplication and in addition you save by the short method.

Following the directions given above, find these products.

a	b	c
1. $364 \times 956 = ?$	$981 \times 429 = ?$	$563 \times 328 = ?$
2. $562 \times 832 = ?$	$872 \times 322 = ?$	$279 \times 252 = ?$
3. $525 \times 164 = ?$	$356 \times 636 = ?$	$648 \times 758 = ?$

3. To multiply any two two-figure numbers whose first digits are the same, use the method shown here.

84	$5 \times 4 = 20$. Write 0. Carry 2.
85	$(5+4) \times 8 + 2 = 74$. Write 4. Carry 7.
7140	$8 \times 8 + 7 = 71$. Write 71.

a	b	c
1. $36 \times 34 = ?$	$28 \times 24 = ?$	$18 \times 16 = ?$
2. $65 \times 63 = ?$	$54 \times 58 = ?$	$2.4 \times 26 = ?$
3. $35 \times 36 = ?$	$6.5 \times 68 = ?$	$8.2 \times 8.1 = ?$
4. $95 \times 97 = ?$	$9.2 \times 9.3 = ?$	$8.7 \times 89 = ?$
5. $48 \times 47 = ?$	$68 \times 68 = ?$	$36 \times 36 = ?$

4. Multiplying any two two-figure numbers.

36	Think $8 \times 6 = 48$. Write 8. Carry 4.
28	Think $(8 \times 3) + (2 \times 6) + 4 = 40$. Write 0. Carry 4.
<u>1008</u>	Think $2 \times 3 + 4 = 10$. Write 10.

Using the method just shown, write these products.

a	b	c
1. $36 \times 43 = ?$	$24 \times 28 = ?$	$85 \times 15 = ?$
2. $63 \times 23 = ?$	$38 \times 92 = ?$	$45 \times 55 = ?$
3. $47 \times 42 = ?$	$19 \times 89 = ?$	$65 \times 84 = ?$
4. $87 \times 92 = ?$	$97 \times 81 = ?$	$97 \times 98 = ?$
5. $54 \times 58 = ?$	$91 \times 16 = ?$	$83 \times 38 = ?$

5. Beginning the multiplication with the first left hand figure of the multiplier.

Many good computers can shorten their work in multiplication because they know how to multiply by beginning with the left hand figure of the multiplier instead of the right hand one.

3464	Multiply by 2. Place the first figure of the partial product under 2.
28	Then multiply by 8 placing the first figure of this partial product under 8.
<u>6928</u>	
27712	
<u>96992</u>	

Using the method shown above find these products.

a	b	c
1. $3487 \times 64 = ?$	$2598 \times 56 = ?$	$9426 \times 95 = ?$
2. $4298 \times 32 = ?$	$543 \times 69 = ?$	$856 \times 89 = ?$
3. $3509 \times 142 = ?$	$2645 \times 198 = ?$	$368 \times 574 = ?$
4. $2563 \times 6.4 = ?$	$786 \times 7.2 = ?$	$605 \times 84.2 = ?$

6. Multiplying without writing the last partial product.

Multiply 384 by 28.

Long Form	Short Form
384	384
28	28
768	768
3072	10752
10752	

Explanation. When you multiply by the last partial multiplier, add the other partial product and write the total product. Thus, $8 \times 4 = 32$. Write 2. Carry 3. $8 \times 8 = 64$. Add 64, 3, and 8 (of first partial product). Think 75. Write 5. Carry 7. $8 \times 3 + 7 + 6 = 37$. Write 7. Carry 3. $3 + 7 = 10$. How much work do you save by the short form?

Using the short form find the following products:

a	b	c
1. $984 \times 35 = ?$	$754 \times 63 = ?$	$896 \times 253 = ?$
2. $528 \times 27 = ?$	$384 \times 21 = ?$	$527 \times 231 = ?$
3. $56 \times 7\frac{3}{4} = ?$	$252 \times 5\frac{1}{2} = ?$	$356 \times 16\frac{1}{2} = ?$
4. $254 \times 5.6 = ?$	$358 \times .66 = ?$	$584 \times 2.56 = ?$

7. Finding approximate products.

In multiplying integers by decimals or decimals by decimals, often the result is accurate enough if you find the product true to ones.

$3757 \times .9 = ?$ Since the product is to be true to ones, think $.9 \times 7 = 6.3$. Drop .3 and carry 6. Think $9 \times 5 + 6 = 51$. Write 1. Carry 5, etc.

$8.75 \times 9.65 = ?$

8.75	Since the product is to be true to ones and there is more than one figure in the multiplier, the partial products should be carried to tenths. Multiply by 9. Think $9 \times .05 = .45$. Carry 4. Drop 5. Think $9 \times .7 + .4 = 6.7$. Write 7. Carry 6. Think $9 \times 8 + 6 = 78$. Write 78. Multiply by .6. You may omit $.6 \times .05$. Why? Think $.6 \times .7 = .42$. Drop 2. Carry .4. Think $.6 \times 8 + 4 = 5.2$. Write 5.2. Multiply by .05. Since $.05 \times$ any number of hundredths cannot produce tenths, drop .75. Think $.05 \times 8 = .4$. Write .4. The complete product is 84. To find out if this answer is true to ones, multiply out in full 8.75 by 9.65 and compare the integral part of your answer with 84.
9.65	
78.7	
5.2	
.4	

Using the plan shown on page 221 find these products.

a	b	c
1. $364 \times .5 = ?$	$846 \times .7 = ?$	$34.8 \times .6 = ?$
2. $84.5 \times .6 = ?$	$32.4 \times .4 = ?$	$85.6 \times .8 = ?$
3. $24.6 \times .56 = ?$	$48.3 \times .15 = ?$	$24.9 \times .26 = ?$
4. $64.6 \times 3.6 = ?$	$42.5 \times 3.2 = ?$	$56.5 \times 8.4 = ?$
5. $27.5 \times 37.5 = ?$	$9.75 \times 8.62 = ?$	$3.54 \times 2.65 = ?$
6. $285 \times 15\% = ?$	$264 \times 21\% = ?$	$16.2 \times 84\% = ?$

8. Computing successive discounts.

The single equivalent discount of two successive discounts is their sum diminished by their product.

With the help of the above rule find the missing numbers in the table below.

List	Discounts	Single Equivalent Disct. in %	Net in \$
1. \$ 840	20 % , 10 %	?	?
2. 2000	$33\frac{1}{3}\%$, 20 %	?	?
3. 900	15 % , 10 %	?	?
4. 600	10 % , 5 %	?	?
5. 300	$\frac{1}{4}$, 10 %	?	?
6. 400	$\frac{1}{2}$, $\frac{1}{3}$	?	?
7. 40	40 % , 15 %	?	?
8. 100	60 % , 20 %	?	?
9. 800	15 % , 15 %	?	?
10. 60	10 % , 10 %	?	?
11. 200	5 % , 5 %	?	?

In Division

1. When the divisor is an easy fractional part of 10 or 100.

To divide a number by $2\frac{1}{2}$ or $3\frac{1}{3}$ is the same as dividing it by 10 and multiplying this quotient by 4 or 3 because $2\frac{1}{2}$ is $\frac{1}{4}$ of 10 and $3\frac{1}{3}$ is $\frac{1}{3}$ of 10.

$16 \div 2\frac{1}{2} = ?$ Think 1.6×4 . Write 6.4. For short, think 4×1.6 .
 $16 \div 3\frac{1}{3} = ?$ Think 1.6×3 . Write 4.8. Think 3×1.6 .

To divide a number by $8\frac{1}{3}$, $12\frac{1}{2}$, $16\frac{2}{3}$, 25, or $33\frac{1}{3}$ is the same as dividing it by 100 and multiplying the quotient by 12, 8, 6, 4, or 3.

$166 \div 8\frac{1}{3} = ?$	Think 12×1.66 .	Write 19.92.
$166 \div 12\frac{1}{2} = ?$	Think 8×1.66 .	Write 13.28.
$166 \div 16\frac{2}{3} = ?$	Think 6×1.66 .	Write 9.96.
$166 \div 25 = ?$	Think 4×1.66 .	Write 6.64.
$166 \div 33\frac{1}{3} = ?$	Think 3×1.66 .	Write 4.98.

Write the answers without sidework.

a	b	c
1. $42 \div 25 = ?$	$242 \div 25 = ?$	$35.2 \div 25 = ?$
2. $96 \div 12\frac{1}{2} = ?$	$356 \div 12\frac{1}{2} = ?$	$84.2 \div 12\frac{1}{2} = ?$
3. $84 \div 16\frac{2}{3} = ?$	$840 \div 16\frac{2}{3} = ?$	$36.4 \div 16\frac{2}{3} = ?$
4. $35 \div 8\frac{1}{3} = ?$	$105 \div 8\frac{1}{3} = ?$	$24.5 \div 8\frac{1}{3} = ?$
5. $63 \div 25 = ?$	$250 \div 12\frac{1}{2} = ?$	$36.8 \div 16\frac{2}{3} = ?$
6. $98 \div 33\frac{1}{3} = ?$	$164 \div 33\frac{1}{3} = ?$	$62.5 \div 8\frac{1}{3} = ?$
7. $98 \div 3\frac{1}{3} = ?$	$164 \div 2\frac{1}{2} = ?$	$78.2 \div 2\frac{1}{2} = ?$

2. When the divisor ends in one or more zeros.

Cancel the zeros in the divisor and move the decimal point in the dividend as many places to the left as there are canceled zeros before you divide.

$$3484 \div 80 = ? \qquad \begin{array}{r} 43.55 \\ 80 \overline{) 348.4} \end{array}$$

Solve these examples by the method just shown.

a	b	c
1. $34867 \div 70 = ?$	$96429 \div 2000 = ?$	$96429 \div 2240 = ?$
2. $26464 \div 80 = ?$	$58346 \div 2000 = ?$	$84670 \div 2240 = ?$
3. $38472 \div 60 = ?$	$67400 \div 440 = ?$	$58380 \div 2000 = ?$

3. Dividing large numbers by large numbers.

In such cases much work may be saved by dropping several of the right hand figures in both the dividend and the divisor. Usually the quotient will be accurate enough for practical purposes if one or two more places are retained in the dividend and divisor than are required in the quotient.

1. Divide 846736 by 52378, carrying the quotient to tenths.

$$\begin{array}{r}
 \text{Full work} \\
 16.1+ \\
 \hline
 52378 \overline{)846736} \\
 \underline{52378} \\
 322956 \\
 \underline{314268} \\
 86880
 \end{array}$$

$$\begin{array}{r}
 \text{Short work} \\
 16.1+ \\
 \hline
 523 \overline{)8467} \\
 \underline{523} \\
 3237 \\
 \underline{3138} \\
 990
 \end{array}$$

Explanation. Inspection shows that the dividend contains the divisor more than 10 times but less than 100 times. Therefore, a quotient to tenths will have 3 figures in it. Hence, the 2 right hand figures in the dividend and divisor may be dropped, which is the equivalent of dividing both terms by 100 and dropping the remainders. This makes the divisor to be used 523 and the dividend 8467.

1. How many operations do you save by the short form?

2. What % of the work is saved by the short form?

2. What % is 2,346,784 of 3,487,564 to tenths of one per cent?

.673 nearly = 67.3% nearly.

$$\begin{array}{r}
 3487 \overline{)2346.000} \\
 \underline{2092 } \\
 253 \\
 \underline{244 } \\
 9
 \end{array}$$

Explanation. Inspection shows the first quotient figure to come in tenth's place. Therefore, there will be 3 quotient figures in the required result. Hence, 4 places were kept in the dividend and the divisor. Divide by the long method to see how much you save by the short form.

Find each of the following ratios to tenths:

1. $348,567 : 319,568$

5. $2,348,654 : 1,952,834$

2. $585,603 : 475,620$

6. $5,602,405 : 4,760,390$

3. $792,465 : 548,378$

7. $9,482,563 : 4,749,608$

4. $258,907 : 198,074$

8. $7,593,896 : 2,593,472$

Problems

1. The estimated value of the 1923 corn crop in the United States at July 1 farm prices was \$2,488,983,005. On July 1, 1922, this value was \$1,798,022,864. Find the per cent of increase to tenths of a per cent.

2. The estimated value of the 1923 wheat crop in the United States at July 1 farm prices was \$780,417,288. July 1, 1922, this value was \$884,505,366. Find the per cent of decrease to tenths of a per cent.

3. The estimated 1923 cotton crop in the United States on July 1 was 11,412,000 bales, and the estimated crop for 1922 was 9,762,000 bales. The price on July 1, 1923, was \$.262 per lb. and on July 1, 1922, it was \$.204 per lb. If the average bale weighs 500 lb., find the per cent of increase of the value of the estimated 1923 crop over that of the 1922 crop.

4. The estimated wheat crop of the world for 1923 was 3,354,132,000 bu. For 1922 it was 3,298,137,000 bu. Find the per cent of increase.

5. The corn crop of the United States for 1923 was 3,021,000,000 bu. For 1922 the crop was 2,890,000,000 bu. What % is the 1923 crop of the 1922 crop?

CHAPTER III

SQUARES AND SQUARE ROOTS

1. The square of a number is the product obtained by using the number twice as a factor.

9 is the square of 3 because $3 \times 3 = 9$. The product may be indicated by writing 2 (called the exponent) a little to the right and above the number to be squared, as $3 \times 3 = 3^2$. Such an expression is read 3 squared, or the square of 3.

2. The square root of a number is one of the two equal factors of that number.

4 is the square root of 16 because 4 is one of the two equal factors of 16. The sign of square root is $\sqrt{}$. The square root of a number may be indicated by writing the sign over the number, as $\sqrt{16}$.

3. Write in a column the squares of all the numbers from 1 to 13, and of all the numbers ending in zero between 10 and 100.

4. Recall the short method of squaring numbers ending in 5.

5. Write in three columns the squares

(1) Of all integers ending in 5 from 15 to 125.

(2) Of all pure decimals of two places ending in 5 from .05 to .95.

(3) Of all mixed decimals of one decimal place ending in 5 from 1.5 to 12.5.

Use this table when you extract square root.

6. Show by example the truth of these statements.

(1) The squares of all numbers between 1 and 10 lie between 1 and 100.

(2) The squares of all numbers between 10 and 100 lie between 100 and 10,000.

(3) The square of a proper fraction is a proper fraction less in value than the fraction squared. $(\frac{3}{4})^2 = \frac{9}{16}$.

(4) The square of a decimal fraction is a decimal fraction with twice as many decimal places as the fraction squared; therefore, an even number of places. $.9^2 = .81$.

(5) The square of a mixed number is a mixed number.

(6) The square root of a decimal fraction has one half as many decimal places as the decimal fraction whose root is extracted. $\sqrt{.49} = .7$.

(7) The square root of a proper fraction has a larger value than the fraction, but it is always less than one.

(8) The square roots of numbers between 100 and 10,000 lie between 10 and 100.

(9) The square roots of numbers between 1 and 100 lie between 1 and 10.

7. Observations.

(1) Perfect squares (numbers whose square root can be found exactly) ending in 25 have their root ending in 5. $\sqrt{4225} = 65$.

(2) Perfect squares ending in 6 have their root ending in 4 or 6. $\sqrt{16} = 4$. $\sqrt{36} = 6$.

(3) Perfect squares ending in 9 have their root ending in 3 or 7. $\sqrt{169} = 13$. $\sqrt{289} = 17$.

(4) Perfect squares ending in 1 have their root ending in 1 or 9. $\sqrt{121} = 11$. $\sqrt{81} = 9$.

(5) Perfect squares ending in 4 have their root ending in 2 or 8. $\sqrt{64} = 8$. $\sqrt{144} = 12$.

(6) No perfect square can end in one zero, 2, 3, 7, or 8.

(7) No perfect square can end in 15, 35, 45, 55, etc.

Extracting Square Root by Long Division

By this method extracting square root consists in finding a divisor which is equal to the quotient when the dividend is the number whose square root is to be extracted.

I. Of perfect squares.

1. Extract the square root of 9801.

$$\begin{array}{r} 99 \\ 99 \overline{)9801} \\ \underline{891} \\ 891 \end{array}$$

The square root must lie between 95 and 100. Why? What is the square of 95? Assuming 9801 to be a perfect square, try 99 as a divisor. Divide 9801 by 99. 99 is the square root of 9801. Why?

In case of perfect squares whose square root does not exceed two places the root can be determined exactly upon the first trial by keeping in mind the observations on page 227 and remembering the squares of the numbers ending in 0 and 5.

2. Extract the square root of 6084.

$$\begin{array}{r} 78 \\ 78 \overline{)6084} \\ \underline{546} \\ 624 \end{array}$$

The root lies between 75 and 80. Why? After you have observed the last digit in 6084, inspection shows the root to be 78, assuming 6084 to be a perfect square. Actual division proves the correctness of the inspection.

3. Extract the square root of 6.76.

$$\begin{array}{r} 2.6 \\ 2.6 \overline{)6.76} \\ \underline{5.2} \\ 1.56 \end{array}$$

Assuming 6.76 to be a perfect square, try 2.6, because $2.5^2 = 6.25$ and $3^2 = 9$.

4. Extract the square root of .6889.

$$\begin{array}{r} .83 \\ .83 \overline{).6889} \\ \underline{.664} \\ .0249 \end{array}$$

Try .83. $.8^2 = .64$ and $.85^2 = .7225$.

The solutions on page 228 represent the 3 types of perfect squares whose roots may be extracted accurately by inspection at the first trial.

5. These numbers are perfect squares. Estimate the square root of each. Then prove your estimate by division.

	a	b	c	d	e
(1)	1369	9409	2809	8281	1156
(2)	1521	1225	7225	9025	2209
(3)	3721	5329	6241	5776	8649
(4)	289	4489	676	961	6561
(5)	7569	3364	3481	4356	1764
(6)	6084	5625	841	361	2025
(7)	3025	6889	4225	2704	529

6. Remembering that the square root of a decimal fraction is a decimal with only one half as many decimal places, find by the method of inspection and long division the square root of each of the following perfect decimal squares.

	a	b	c	d	e
(1)	.3025	.0484	.7225	.9025	.9409
(2)	.4225	.6241	.5625	.2025	.3364
(3)	.4489	.0625	.2704	.3136	.1225
(4)	.1681	.2809	.6084	.3721	.5329

7. The following mixed decimals are perfect squares. Find their square root. Remember the decimal point. Check your answer.

	a	b	c	d
(1)	42.25	30.25	6.25	92.16
(2)	1.96	1.69	90.25	51.84
(3)	10.89	32.49	5.29	86.49
(4)	15.21	29.16	21.16	75.69

II. Extracting the square root of imperfect squares.

People who must often know the square root of imperfect squares use a table like the one on page 399. If a table is not convenient, use the method of inspection and long division as shown in example 1.

1. Extract the square root of 23.

$$\begin{array}{r} 4.79+ \\ 4.8 \overline{)23.00} \\ \underline{19.2} \\ 3.80 \\ \underline{3.36} \\ .440 \end{array}$$

Inspection shows the root to lie between 4.5 and 5. What is the square of 4.5? The root is nearer 5 than 4.5. How may you know this? Try 4.8. Compare the quotient with the divisor. Have you found the square root? For practical purposes a square root carried to tenths is accurate enough in ordinary work. If you find your first estimate not accurate enough, use it to help you make another one, as

shown in the next example.

2. Extract the square root of 1800 to tenths.

$$\begin{array}{r} 42.6 \\ 42.2 \overline{)1800.00} \\ \underline{1688} \\ 112.0 \\ \underline{84.4} \\ 27.60 \end{array}$$

The square root lies between 40 and 45. It is larger than 42 and less than 42.5. How may we know this? What is the middle number between 1600 and 2025? Try 42.2 as the root.

Compare the quotient with the divisor. Try their average (42.4) as the next divisor.

$$\begin{array}{r} 42.4+ \\ 42.4 \overline{)1800.00} \\ \underline{1696} \\ 104.0 \\ \underline{84.8} \\ 19.20 \\ \underline{16.96} \\ .240 \end{array}$$

The square root is 42.4+ true to tenths. If the root should be required to two decimal places, divide again. Compare the quotient with the divisor and proceed as before.

3. The sign $\sqrt{\quad}$ means extract the square root. In integers and mixed decimals, carry the root to tenths; in pure decimals, to hundredths. Remember the decimal point. Many pupils have estimated correctly on the first trial in most examples of this list. Check your answer.

	a	b	c	d	e
(1)	$\sqrt{34}=?$	$\sqrt{9125}=?$	$\sqrt{.9}=?$	$\sqrt{.7}=?$	$\sqrt{1525}=?$
(2)	$\sqrt{3.4}=?$	$\sqrt{7325}=?$	$\sqrt{.2250}=?$	$\sqrt{.4}=?$	$\sqrt{3000}=?$
(3)	$\sqrt{79}=?$	$\sqrt{125}=?$	$\sqrt{.036}=?$	$\sqrt{90}=?$	$\sqrt{285}=?$
(4)	$\sqrt{70}=?$	$\sqrt{42.9}=?$	$\sqrt{.5648}=?$	$\sqrt{36.8}=?$	$\sqrt{25.6}=?$

III. Extracting the square root of common fractions and mixed numbers.

1. If the fraction in its lowest terms, or the mixed number when it is reduced to an improper fraction, has perfect squares for both numerator and denominator, extract the square root of both numerator and denominator.

$$\sqrt{\frac{9}{16}} = \frac{3}{4} \qquad \sqrt{2\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} \qquad \sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

2. In all other cases involving fractions or mixed numbers, reduce to decimals and then extract the square root to the required number of places.

$$\sqrt{2\frac{1}{3}} = \sqrt{2.33+} = 1.5+ \sqrt{\frac{3}{4}} = \sqrt{.75} = .86+ \sqrt{\frac{8}{9}} = \sqrt{.8888+} = .94+$$

Examples

HINT.—Study your example before you make the estimate.

	a	b	c	d	e
1.	$\sqrt{\frac{9}{25}}=?$	$\sqrt{1\frac{7}{9}}=?$	$\sqrt{2.4}=?$	$\sqrt{\frac{3}{8}}=?$	$\sqrt{6\frac{2}{3}}=?$
2.	$\sqrt{\frac{4}{9}}=?$	$\sqrt{30\frac{1}{4}}=?$	$\sqrt{12.6}=?$	$\sqrt{\frac{2}{3}}=?$	$\sqrt{7\frac{1}{5}}=?$
3.	$\sqrt{\frac{64}{25}}=?$	$\sqrt{3\frac{1}{16}}=?$	$\sqrt{2\frac{3}{4}}=?$	$\sqrt{2\frac{5}{8}}=?$	$\sqrt{18\frac{3}{5}}=?$

If you wish to discover your comprehension and computation skill in square root, try the next two tests.

I

**Comprehension of the Relation between
a Number and its Square Root**

Time—10 Minutes

1. Give a case in which the square root of a number is equal to the number.

2. Complete the next sentences by using the proper one of the following terms: *smaller, larger, much smaller, much larger, almost equal*.

- a. The square root of a large integer is_____.
- b. The square root of a small integer is_____.
- c. The square root of a large proper fraction is_____.
- d. The square root of a small proper fraction is_____.
- e. The square root of a small decimal fraction is_____.
- f. The square root of a large decimal fraction is_____.
- g. The square root of a very small proper fraction is_____.

3. State an example for each of your answers in No. 2.

II

A Race in Extracting Square Root

12 Right in 15 Minutes

When the square is a decimal fraction, carry the square root to two decimal places; in all others to one decimal place.

a	b	c	d	e
1. 56.25	.8924	7225	96	149
2. 90.25	.1324	8281	75	130
3. 8.4	.0324	5776	35	442

CHAPTER IV

USING SQUARES AND SQUARE ROOTS

The Right Triangle

1. Draw a right triangle whose base is 4 in. and whose altitude is 3 in. Measure the long side, called the hypotenuse. The figure you drew is called the 3-4-5 (Read three four five) triangle. It occurs frequently in problems, and therefore should be carefully studied.

2. Erect squares on the three sides of the triangle you drew in problem 1. What is the relation as regards area between the large square and the other two?

3. Draw to some convenient scale the right triangle whose sides are 9 in., 12 in., and 15 in. Erect squares as you did in problem 2. What is the relation between the largest square and the other two?

4. The square on the hypotenuse of a certain right triangle contains 100 sq. in. The square on the altitude contains 36 sq. in. What is the area of the square on the base? How long is the base?

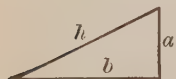
5. Beginning at one corner of the floor of your school-room measure on the floor along one wall exactly 6 feet, making a mark at the end of the 6-foot line. Then from the same corner measure on the floor along the other wall exactly 8 feet, making a mark at the end of the 8-foot line. Now connect the two points with a stiff cord and measure the length of the cord between the two points. What do you find? What is the name of this line?

The truth illustrated in the previous problems may be called the "Rule of the Right Triangle." It may be stated in this way,

The square on the hypotenuse of any right triangle is equal to the sum of the squares on the two other sides.

Expressing the Rule of the Right Triangle in a Formula

1. A formula is a rule stated in letters.



2. In right triangles, h stands for the hypotenuse, a stands for the altitude, and b for the base. See the figure.

When used in formulas, these letters stand for abstract numbers. If in a given formula h represents feet, then a and b must also represent feet.

3. The rule of the right triangle may be expressed in these ways. Tell the meaning of each from the figure.

$$(1) h^2 = a^2 + b^2; h = \sqrt{a^2 + b^2}.$$

$$(2) a^2 = h^2 - b^2; a = \sqrt{h^2 - b^2}.$$

$$(3) b^2 = h^2 - a^2; b = \sqrt{h^2 - a^2}.$$

4. Find h when $a=4$ and $b=3$.

$$h = \sqrt{4^2 + 3^2}. \quad h = \sqrt{25} \text{ or } 5.$$

Solve these right triangles with the formula.

(1) Find h when $a=20$ and $b=21$; when $b=33$ and $a=56$.

(2) Find a when $h=53$ and $b=45$; when $h=89$ and $b=80$.

(3) Find b when $a=36$ and $h=85$; when $a=44$ and $h=125$.

One of the triangles in the next list is not a right triangle. Find it and prove your answer.

(1) $a = 5, b = 12, h = 13$.

(3) $a = 12, b = 12, h = 15$.

(2) $a = 30, b = 40, h = 50$.

(4) $a = 15, b = 36, h = 39$.

1. From the 3-4-5 triangle in the figure find by inspection the base and altitude of each triangle whose hypotenuse is shown.

2. How do the triangles in the figure resemble each other? How are they different?

3. Write the sides of three other triangles which belong to this group.

4. By remembering the 3-4-5 triangle, you can often solve problems by inspection (without use of pencil). Try this on the following right triangles.

5. Base, 21 feet; altitude, 28 feet. Find hypotenuse.

HINT.—By what must you multiply the dimensions of the 3-4-5 triangle to make a 21-28-(x) triangle?

6. Base, 24 in.; altitude, 32 in. Find hypotenuse.

7. Hypotenuse, 45 feet; altitude, 36 feet. Find the base.

8. Hypotenuse, $1\frac{1}{4}$ inches; base, 1 inch. Find the altitude.

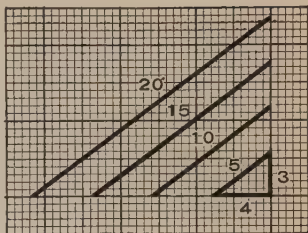
9. Find the diagonal of a rectangle 42 ft. wide and 56 ft. long.

10. The diagonal of a rectangular field is 100 rods; its width is 60 rods. Find the length.

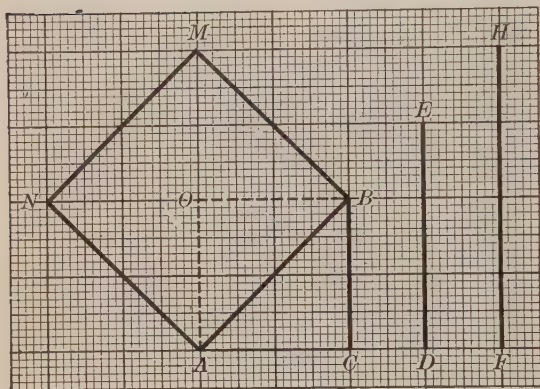
11. If your schoolroom is 32 ft. by 24 ft., find the diagonal of the ceiling. Pupils who know the 3-4-5 triangle do not need a pencil in this problem.

12. The diagonal of a rectangular lot is 115 ft. Its length is 92 ft. Find its width.

HINT.—23 is a common factor of 92 and 115. Therefore the triangle reduces to a 3-4-5 triangle.



The Isosceles Right Triangle



1. ABC is an isosceles right triangle. Why?

2. What relation exists between the square on the base and the one on the altitude?

3. What relation is there

between the square on the hypotenuse and the square on either one of the equal sides?

4. AB is also the diagonal of the square $AOBC$.

5. If the diagonal of the square in the figure is 10 feet, can you find the area of the square without finding the length of one side of the square? How? What is the area of the square $ABMN$? Of $AOBC$?

6. From the figure and your work write a rule for finding the area of any square when you know its diagonal.

7. DE represents the height of a tree from the ground to the first limb. How can you find the length of DE by using the triangle ABC ? If HF is a telephone pole, can you find its height by means of ABC ? Do it.

8. Some boys wished to know how they might measure the height of trees, towers, telephone poles, etc. This is the way their teacher told them to proceed.

I. Constructing the Apparatus

(1) Cut from heavy cardboard, or a thin board, an isosceles right triangle such as ABC in the figure on page 236.

(2) Mount this triangle on a level stand of convenient height as shown in the figure below.

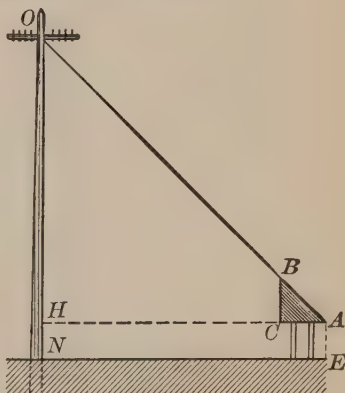
II. Using the Apparatus

Problem. How high from the ground is the cross piece of the telephone pole near school?

(1) Set the apparatus on a level at such a distance that the line AB is in line with O , the upper extremity of the vertical distance to be measured. See figure.

(2) The line AH is horizontal and at right angles to the telephone pole in the figure. Why?

(3) The triangle AOH is isosceles right. Why?

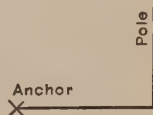


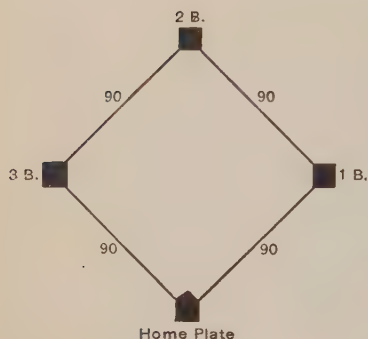
III. Computing the Result

(1) The distance of the cross piece at O from the ground at $N = AH + AE$. Why?

(2) How can you find the length of AH ?

10. How long must a wire be to reach from the top of a pole 57 feet high to an anchor in the ground 76 feet from the foot of the pole?





11. A baseball diamond is 90 feet square.

(1) How far must the catcher, standing at the home plate, throw (measured on the ground) to reach the man at second base? Carry result to one decimal place.

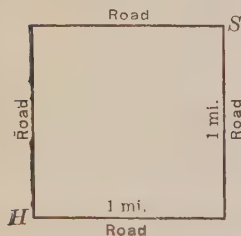
(2) How far is it from first base to third base?

(3) If the center fielder stands 100 feet directly behind second base, how far is he from the home plate?

(4) How far is it from the center of the diamond to third base?

(5) How far measured on the ground must a player bat a ball to make it strike 20 ft. to the left of the center fielder who stands 100 feet directly behind second base?

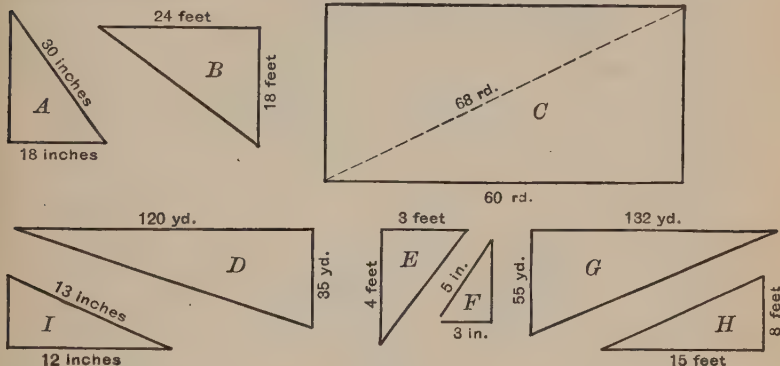
12. If a boy living at H , going to school at S , can go by way of the diagonal instead of over the roads, how much can he save in a school year of 200 days? If you express your answer in miles, carry it to 2 decimal places.



13. A girl walks $3\frac{1}{2}$ mi. due south and then $3\frac{1}{2}$ mi. due west. How far measured in a straight line is she from the starting point?

14. Two boys riding bicycles meet at a cross road. One is riding due east at 8 miles an hour, the other is going due south at 6 miles an hour. How far apart are they in $2\frac{1}{2}$ hours?

1. Find the length of a guy wire one end of which is attached to a 60-foot smoke stack 6 feet from the top. The other end is fastened at the ground 50 feet from the base of the stack. Allow 2 feet for slack wire.

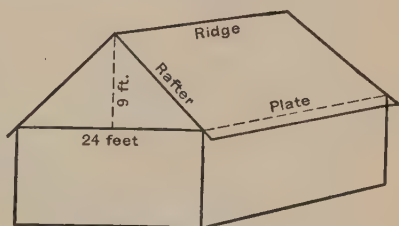


2. Find the unknown side and area of each of the above figures. Find the unknown side by inspection.

3. Study this figure.

(1) How long must the rafters be to reach the plate?

(2) How long if they extend 18 in. beyond the plates?



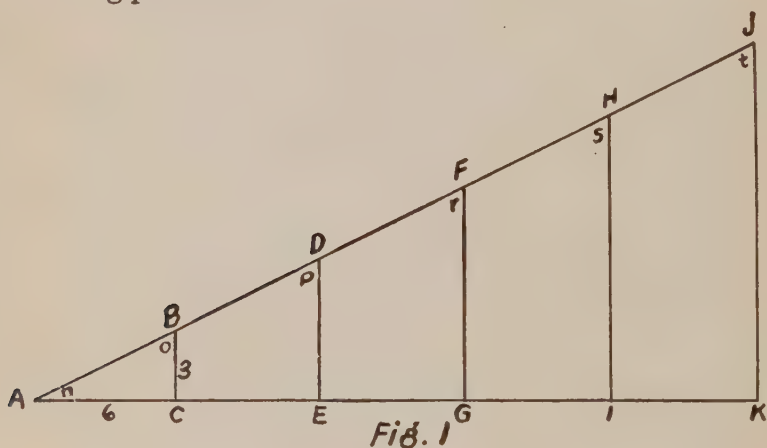
(3) Find the area of the roof if its length is 49 ft.?

4. How much lumber one inch thick will be required for the siding of the barn shown in the figure if it is 48 ft. long and 16 ft. high to the plate and if $\frac{1}{2}\%$ of that required for the gable ends must be added for waste in cutting? What will be the cost at \$40 per M?

CHAPTER V

SIMILAR TRIANGLES

In your work with right triangles in the last chapter you learned how the right isosceles triangle may be used to find the length of unknown heights and distances. In this chapter you will learn another way of using triangles in solving problems.



1. In figure 1 there are five right triangles. Read them.
2. Which angle is common to all the triangles?
3. Angle o is the complement of $\angle n$. Why?
4. Angle p is also the complement of $\angle n$. Why?
5. Then $\angle o = \angle p$. Why?
6. Angles o, p, r, s , and t are all equal to each other. Why?
7. What can you say of the angles of the five triangles?
8. Why do these triangles have the same shape?

Triangles whose sides are unequal, but whose angles are equal each to each (corresponding angles equal), are called similar triangles.

1. If the altitude of $\triangle ABC$ in figure 1 is 3, find by measurement the altitude of each of the other triangles.

2. If the base of $\triangle ABC$ is 6, find by measurement the base of each of the other triangles?

3. What is the ratio of the base of $\triangle ABC$ to the base of $\triangle ADE$?

4. What is the ratio of the base of $\triangle ABC$ to the base of each of the other triangles? Record your answer.

5. What is the ratio of the altitude of $\triangle ABC$ to the altitude of $\triangle ADE$?

6. What is the ratio of the altitude of $\triangle ABC$ to the altitude of each of the other triangles? Record your answer.

7. If the altitude of $\triangle ABC$ is 3, find the hypotenuse by measuring to scale.

8. Compare the value of the hypotenuse in No. 7 with the value you get by the Rule of the Right Triangle.

9. Find the ratio of the hypotenuse of $\triangle ABC$ to the hypotenuse of each of the other triangles. Record your answer.

10. Compare the ratios in No. 9 with those you found in Nos. 4 and 6.

Triangles whose corresponding sides have the same ratio are said to be similar.

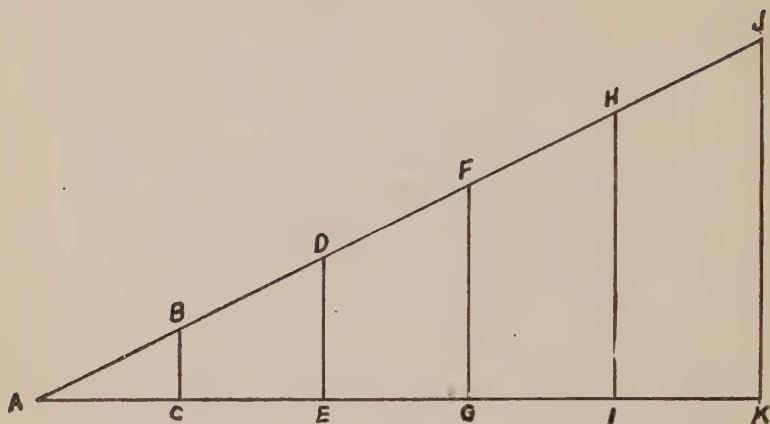


Fig. 2

1. Measure the triangles in figure 2 to determine if they are similar.

2. If the base and altitude of $\triangle ABC$ are 10 and 5 and and base of $\triangle ADE$ is 15, what is the altitude of $\triangle ADE$?

3. If in $\triangle ABC$ AC is 5 in. and BC is $2\frac{1}{2}$ in. and if AE is 15 ft. in $\triangle ADE$, what is DE ?

4. If the ratio of the altitude to the base in one of two or more similar triangles is $\frac{3}{4}$, what is the ratio of the altitude to the base in each of the other triangles?

5. Measure the triangles in figure 3 to determine if they are similar.

6. If you call the base of $\triangle 123$ in figure 3 n feet, what is the length of the bases and altitudes of the other triangles?

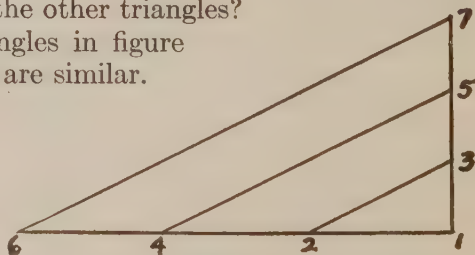


Fig. 3

1. A class wished to determine the width of this stream at OP without crossing it.

They measured up stream to an object R . They continued this line to a point S , where they sighted a tree M , in a line perpendicular to the stream line at S . They measured along SM to a point T , which was in line with R and P .

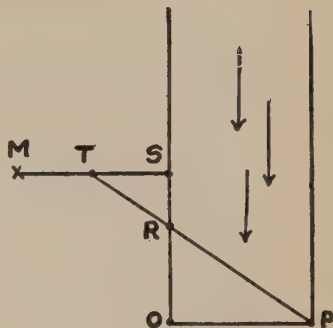


Fig. 4

They then had two right triangles which are similar. Which are the corresponding sides?

They also knew the lengths of three sides of these triangles. Which three?

2. If $OR = 50$ ft., $RS = 25$ ft., and $RT = 37\frac{1}{2}$ ft., find OP .

3. When RS is $\frac{3}{4}$ of OR , then ST is what part of OP ?

4. If CD in figure 5 is 2 rd., CE is 5 rd., and AD is 10 rd., find AB .

5. Having found the length of AB , how can you find the length BF from figure 5?

6. How would you find the length of BF in the field after knowing AB ?

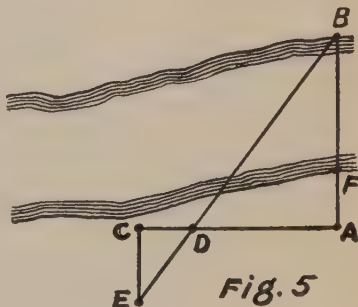
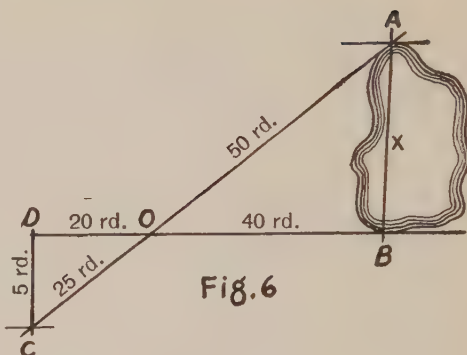


Fig. 5

7. If in figure 5 the ratio of CD to AD were $\frac{2}{3}$, what would be the ratio of CE to AB ? Of ED to DB ?

8. Figure 6 shows how the length of a pond may be determined by indirect measurement. Study the figure to find out how it is constructed. Is it necessary to make as many measurements as are shown in the figure? Why?



9. Why are the two triangles similar? What are the corresponding sides?

10. What is the ratio of DO to OB ? What must be the ratio of CD to AB ? How long is AB ?

11. Knowing OB and AB , in what two ways may you find the distance OA from the figure?



Fig. 7

12. A boy 5 ft. 6 in. tall casts a shadow 8 ft. 3 in. long. How high is a tree that casts a shadow 40 ft. long at the same time?

13. At 10 o'clock a certain 6-foot post casts a 4-foot shadow. How high is the building which casts a 50-foot shadow at the same time and place?

To measure the height of this little tree two pupils placed a 6-foot stake between the tree and the drinking fountain so that the top of the stake was in the line of sight with the outer edge



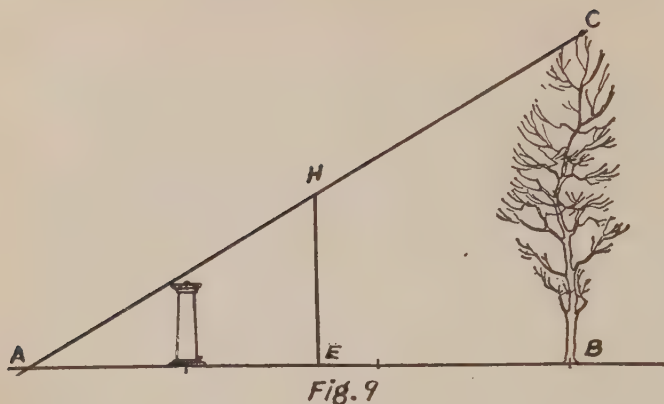
Fig. 8

of the top of the fountain and the top of the tree. They knew that the fountain was 3 feet high.

1. Study the figure and then tell how a knowledge of similar triangles would help the pupils solve the problem in the field.

2. In the figure the fountain is $\frac{7}{16}$ in. high. Using your knowledge of scale drawing and of similar triangles find

- (1) The height of the tree.
- (2) The distance from the center of the fountain to the center of the tree trunk at the base.
- (3) The diameter of the fountain base.
- (4) The distance of the stake from the tree.



3. When the sun's rays have the direction of the hypotenuse of $\triangle ABC$, what is the length of the shadow of the little tree? What is the length of the shadow of the fountain? Of the 6-foot stake EH ?

4. When the shadow of the little tree is 16 ft., how long is the shadow of the fountain?

5. When the shadow of the fountain is $2\frac{1}{2}$ ft., what is the shadow of the tree?

6. If you know the length of the shadow of a tree and your own height and shadow, how can you find the height of the tree?

Measuring Triangles and Trapezoids Drawn to Scale

Mr. Banks bought the land between Clay St. and Adams St. making the triangle MNO . He later laid off the blocks and made cross streets as shown in the map on the next page. This map is drawn to the scale $\frac{1}{16}$ in. = 50 ft.

1. What is the perimeter of $\triangle MRP$? Of $\triangle MON$? See next page.



Fig. 10. ($\frac{1}{6}$ in = 50 ft.)

2. How many acres in the $\triangle MON$?
3. Why are $\triangle s MRP$ and MON similar?
4. What is the width of Adams St.? Of each of the cross streets?
5. What kind of figure is block g ? What are its dimensions?
6. Block f was sold at 25¢ a square foot. How much did it bring?
7. How many square yards of paving in Adams St. from M to O ?
8. Knowing MO and ON in feet, how can you find NM without measuring? Do it. Then compare your answer with the measured distance.
9. Mr. Jones, who owned block c , claimed to have a third more land than Mr. Smith, who owned block b . How nearly was he right?
10. Miss Ross, the owner of block d , said she had twice as much land as the owner of b . How nearly was she right?

HINT.—Compare the average width of d with that of b .

Problems

Unless already shown in the problem, you should draw a figure for every mensuration problem before solving it.

1. How much is saved by walking the diagonal of a rectangular field 80 rods by 40 rods instead of following the length and width?

2. How many feet of wire in a 5-wire fence around a square farm containing 40 acres?

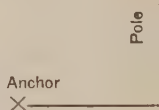
3. Find the altitude and area of the equilateral triangle shown in the figure if each side is 10 ft.



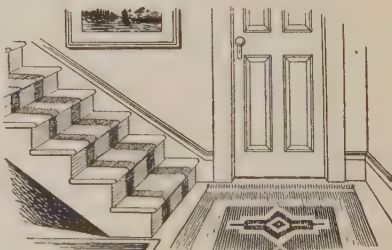
4. In an isosceles triangle the base is 8 inches and each of the other sides is 12 inches. Find the area.

5. Will a 20-foot ladder with its base 5 ft. from the building reach a window 19 ft. above the ground? How do you know? Draw the figure.

6. How long must a wire be to reach from the top of a pole 57 feet high to an anchor in the ground 76 feet from the foot of the pole?



7. The distance from the first floor of a certain house to the second floor is 10 ft. The rise of each step in a stairway is $7\frac{1}{2}$ in. and the tread is 10 in. wide. How many steps in the stairway? How long must the



timbers be from which were cut the notched boards which support the steps at the sides?

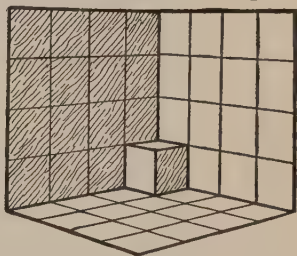
CHAPTER VI

COMPUTING THE CONTENTS AND SURFACES OF SOLIDS

Rectangular Prisms

1. A rectangular prism is a solid which has length, breadth, and thickness, and six rectangular faces. What is a rectangle? Point out objects in your schoolroom representing rectangular solids.

2. The figure on this page represents a box 4 in. long, 4 in. wide, and 4 in. high, of which the front sides and top have been removed. How many cubic inches in its volume or contents?



The volume of a rectangular solid is the volume of its base, one unit high, multiplied by the number of such units in the thickness.

3. Find the volume of a rectangular prism, 6 feet long, 4 feet wide, and 3 feet thick.

The volume = $24 \text{ cu. ft. (Why?)} \times 3 \text{ (Why?)} = 72 \text{ cu. ft.}$

4. How many cubic inches are there in a block 8 in. $\times$ 5 in. $\times$ 4 in.?

5. A solid is 8 inches $\times$ 4 inches $\times$ 2 inches. Find its volume. Do you know any such solids?

6. How many faces are there in the solid in problem 5? The bottom and top faces are called the upper base and the

lower base. Find the area of these bases. The other faces (how many?) are called lateral faces. Find the surface (called lateral surface) of these faces.

**The Entire Surface of a Solid Equals the Surface of the
Bases Plus the Lateral Surface**

7. Find the entire surface of the solid in problem 5.
8. How many cubic feet of air space in your schoolroom?
9. How many cubic feet of air space per pupil in problem 8?
10. A bin is 14 ft. $\times$ 12 ft. $\times$ 6 ft. At \$2.00 per bushel find the value of the wheat in this bin when one-half full.
11. How many tons of hard coal are there in a bin, 10 ft. $\times$ 12 ft. $\times$ 6 ft., if there are $34\frac{1}{2}$ cu. ft. in a ton? How many bushels in the bin? (A bushel of coal weighs 80 lb.)
12. Ask your janitor to give you the approximate dimensions in feet of your school coal bin. Then find how many tons of soft coal it will hold when $\frac{3}{4}$ full.
13. If the rainfall for a month is 3 inches, how much water falls on a boy's thrift garden, 100 feet by 40 feet?
14. In Columbus, O., the snow is sometimes 6 inches deep on the level. How many cubic feet does a boy shovel if he clears a walk 100 feet long and 4 feet wide?
15. The inside dimensions of a certain Pennsylvania steel freight car are 40 ft. 1 in. $\times$ 8 ft. 10 in. wide $\times$ 9 ft. 1 in. high. Find the capacity to the nearest cubic foot.
16. How many bushels of wheat may be placed in the car in problem 15 if it is filled to a depth of 5 feet?
17. Find and solve other problems, not in books, involving volumes and surfaces of rectangular solids.

Finding the Volume of Cylinders

1. A cylinder is a solid with equal parallel circles for bases and with a uniform diameter.

2. Name 5 objects which may be considered cylinders for all practical purposes.

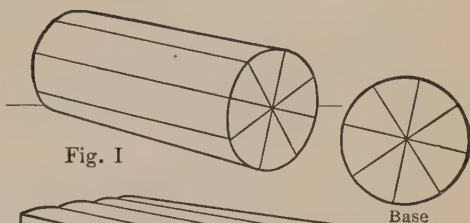


Fig. I

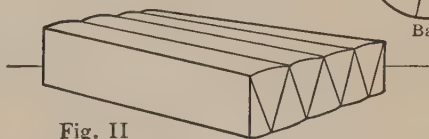


Fig. II

3. How are figures I and II alike? How are they different? Name each figure.

4. Figure II is made from figure I. The dimensions of the base of figure II are the radius and one-half the circumference of the base of figure I.

5. How do you find the volume of figure II?

6. If the radius of figure I is 2 inches and its length is 12 inches, what is the volume of figure II? What is the volume of figure I?

The volume of a cylinder is equal to the volume of the base one unit high multiplied by the number of such units in the height.

7. Find the volume of a cylindrical tomato can whose radius is 2 inches and whose height is $4\frac{1}{2}$ inches.

$$\text{Volume} = (\pi \times 2 \times 2 \text{ cu. in.}) \times 4\frac{1}{2}.$$

$$= 4\pi \text{ cu. in.} \times 9/2 = 18\pi \text{ cu. in.}$$

Explain the above work. Estimate the value of 18π by giving π a value of $3\frac{1}{2}$.

8. How many cubic inches of water in a cylindrical tumbler 3 in. across the top and $3\frac{1}{2}$ in. deep, if it is full?

9. A cylindrical sugar box is 8 inches across and 5 inches deep. Find its capacity.

10. A certain size of Karo syrup cans is $5\frac{1}{2}$ in. deep by $4\frac{1}{2}$ in. in diameter. How much does the can lack of holding one-half gallon?

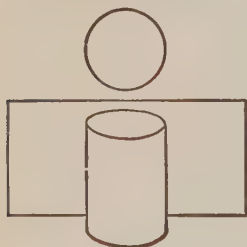
11. A cylindrical fish bowl is 15 in. across the top and 18 in. deep. How many gallons of water are there in it when it is $\frac{2}{3}$ full?

12. Measure 5 cylindrical objects at home and compute the volume of each.

Finding the Surface of Cylinders

1. Why do we need to know how to find the surface of cylinders?

2. Make a problem in which you need to know how to find the surface of a cylinder in order to solve your problem.



3. The figure shows the shape and size of a sheet of paper required to cover the curved surface (also called convex surface) of a cylinder which is $\frac{5}{8}$ in. high and $\frac{7}{16}$ in. across the top. What are the dimensions of the paper?

4. What is the area of the paper?

5. How do you find the curved surface of a cylinder?

6. How do you find the entire surface of a cylinder?

7. A steam pipe is 30 feet long and 4 inches in diameter. What is its heating surface?

8. How much will it cost to paint a cylindrical smoke-stack 40 ft. high and 3 ft. in diameter at \$2.00 per 100 square feet?

9. How many square feet of sheet iron in a furnace pipe whose length is 15 feet and whose diameter is 9 inches, if $\frac{1}{2}$ inch is allowed for lap along the length?

Cylinder Problems

1. How many thousand cubic feet of gas are there in a cylindrical tank 24 feet in diameter and 30 feet high?

2. What is the value of the gas in problem 1 at 80¢ per 1000 cubic feet?

3. How much sheet iron, if $\frac{1}{2}$ inch is allowed for the lap, will be required to make four pieces of stove pipe each 30 inches long and 5 inches in diameter?

4. How much sheet iron in a cylindrical metal cask 4 feet high and 30 inches in diameter if a strip 1 inch wide is allowed along the height for lap?

5. Find the value at 25¢ a gallon of the gasoline in the above cask when it is $\frac{3}{4}$ full.

6. A man wishes to make a cylindrical half bushel measure with a diameter of 12 inches. How deep must it be?

7. A wagon for watering streets has a cylindrical tank 3 ft. in diameter and 8 ft. long. How many gallons will it hold? Count $7\frac{1}{2}$ gal. to the cu. ft.

8. How many gallons of water in a cylindrical cistern 10 feet deep and 4 feet in diameter? (Use $7\frac{1}{2}$ gallons to the cubic foot, also 231 cubic inches to the gallon. How much is the difference in the two answers?)

Practical Problems about Solids

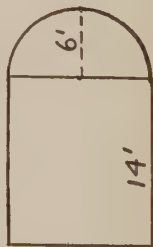
1. A boy wished to build a rectangular coal bin in a basement which is 7 ft. from floor to ceiling. The bin is to hold 6 tons of soft coal when it lacks one foot of being full. Find to the nearest one-half foot the length and width of the bin.

2. How many tons of anthracite coal can be put into the bin in problem 1 if the bin is filled to within 6 inches of the top? Compute the value of this coal at 50¢ a bushel.

3. At the entrance to a certain school yard there are two rectangular granite pillars each 5 ft. high, having a square base each side of which is 20 inches. Find the weight in tons if a cubic foot weighs 165 lb.

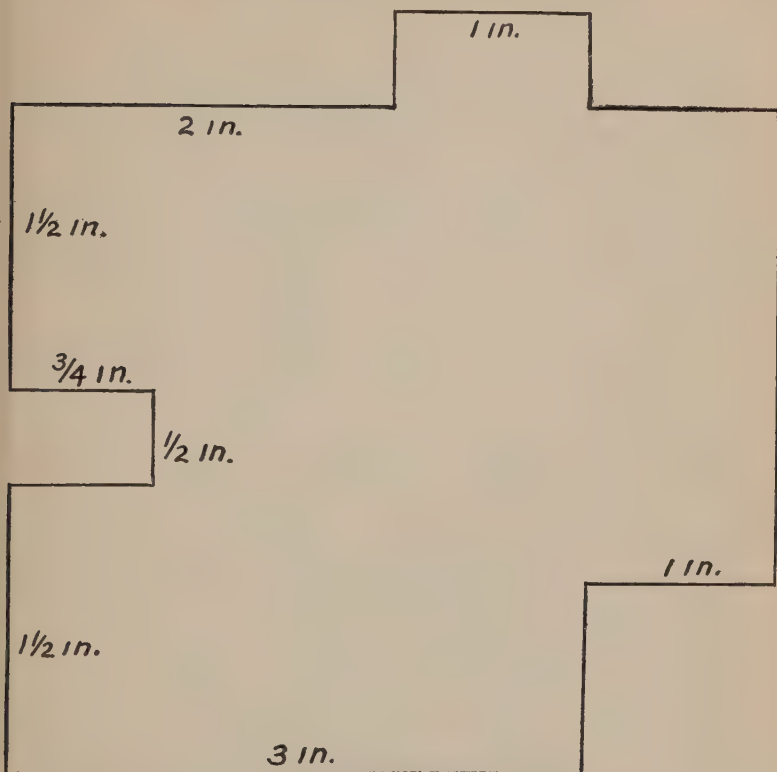
4. A circular pond has a diameter of 200 ft. When the ice is uniformly 18 inches thick, how many tons are there on the pond. Ice weighs $57\frac{1}{2}$ lb. to the cu. ft.

5. The mouth of a tunnel is a rectangle surmounted by a semi-circle with dimensions as shown in the figure. How many cubic yards of earth and rock were removed in excavating the tunnel if it is 300 feet long?



6. Find the weight of an iron rod $\frac{1}{2}$ inch in diameter and 24 feet long if iron weighs 450 lb. per cubic foot.

7. The drive wheel of a steam engine makes 144 revolutions a minute. The cylinder in the steam chest is 16 in. long and 9 in. in diameter and is filled with steam twice in every revolution. How many cubic feet of steam are used in a minute?



Scale.—1 in. = 8 ft.

The figure shows the ground plan of a cellar.

1. Draw it to the scale shown on this page.

2. Divide the figure into rectangles and find the area of each. At 33¢ a sq. ft. find the cost of the concrete floor.

3. In excavating, ground was removed to a depth of 4 ft. How many loads of earth, each 1 cubic yd., were removed?

CHAPTER VII

LEARNING AND USING FORMULAS IN PROBLEM SOLVING

How a formula states a rule or principle in a short way is shown in the comparison below. The rule is written out in full. The corresponding formula is placed in the second column.

Rule	Formula
1. The perimeter of a square is 4 times the length of one side.	1. $P_{\square} = 4s$
2. The circumference of a circle is π times the diameter.	2. $C_{\odot} = \pi D$
3. The area of a triangle is equal to that of a rectangle whose base is the base of the triangle and whose altitude is one-half the altitude of the triangle.	3. $A_{\triangle} = b \times \frac{a}{2}$
4. The area of a circle is equal to that of a rectangle whose base is one-half of the circumference of the circle and whose altitude is the radius of the circle.	4. $A_{\odot} = \frac{C}{2} \times R$

Writing Formulas

1. Write the formula which shows the relation between the diameter and the radius of a circle.

2. If s represents the side of a square, write the formula which tells how to find the area of a square.

3. Calling s p the selling price, c the cost, and g the gain, write a formula which shows how to find the gain when you know the selling price and the cost.

4. Calling distance traveled d , speed per hr. s , and the time in hours t , write a formula which shows the distance a train has run when you know the average speed per hour and the time in hours.

Interpreting Formulas

Write in good English the corresponding rule for each of these formulas.

- | | | |
|-----------------------------|-------------------------------|---|
| 1. $D \odot = C \div \pi$. | 2. $g = sp - c$. | 3. $A \triangle = a \times \frac{b}{2}$. |
| 4. $A \odot = \pi R^2$. | 5. $A \square = a \times b$. | 6. $A \odot = \frac{\pi D^2}{4}$. |

Using Formulas in Problem Solving

1. The formula, $A = \pi R^2$, tells you how to solve two types of circle problems: one direct problem and one indirect problem.

The direct problem is, Find A when you know R .

The indirect problem is, Find R when you know A .

In the indirect problem you should think about the formula, $A = \pi R^2$, in this way. "I know π , also A which is the product of R^2 by π . To find R^2 I must divide A , whose value is known, by π ." How do you find R after finding the value of R^2 ?

2. The general rule for interest is "Multiply the principal by the rate by the number of years."

The formula for this rule is $I = P \times R \times T$.

The formula says I is the product of three factors, P , R , and T . If you know these three factors, you readily find I . This is the direct problem in interest and the most important one.

There are three indirect problems in interest, which can be solved with the formula. One of these problems is: Given I (the product of three factors), P and R (two factors), find T (the third factor).

In general, you find the third factor by dividing the product of the three factors by the product of the two given factors. In the interest problem you would find T (in years) by dividing I (all the interest) by $P \times R$ (the interest for one year).

You have now been shown how the formula, $I = P \times R \times T$, helps to solve two kinds of interest problems.

There are two other interest problems which can be solved from the above formula. Can you tell what they are?

3. In buying and selling problems you have learned that the loss is the *difference* between the *cost* and the *selling price*. This truth may be expressed by the formula $l = c - sp$.

By means of this formula you can solve three types of buying and selling problems: one direct problem and two indirect problems.

The direct problem is: Find l , given sp and c .

One of the indirect problems is: Find c , given l and sp

HINT.—How do you find the subtrahend when you know the..... and the.....?

By this time you doubtless wonder, "How many different problems are there for any given formula?" There is always one direct problem, which is the problem out of which the formula grew. The formula, $I = P \times R \times T$, grew out of this problem: Given the principal, the rate, and the time in years, find the interest. What was the problem which gave rise to the formula $l = c - sp$? There are as many indirect problems in a formula as there are letters representing different numbers on the right of the equality sign.

It follows then that with any given formula you can solve as many different types of problems as there are different letters (terms) in the formula provided that at any given time all the terms but one are known.

Relation of Formulas to the Fundamental Processes

A formula in arithmetic is a general rule expressed in terms of one of the four fundamental processes. The formula for the area of a circle, $A = \pi R^2$, and the formula for interest, $I = P \times R \times T$, are multiplication formulas, no matter which type of problem is to be solved. The thinking in these formulas should be in terms of product and factors (multiplier and multiplicand) just as in the buying and selling formula, $l = c - sp$, the indicated thinking for the three types of problems should be in terms of remainder, minuend, and subtrahend.

Formulas and the Four Fundamental Processes

If you have used properly the equation in your problem solving heretofore, you have found that many problems

could be translated into an equation of one of the four fundamental processes. It is also true that the direct and indirect problems of each fundamental process can be expressed as formulas, which are equations with letters representing numbers.

Where a term in a formula is missing as indicated by the ?, name it and tell how it may be found.

Addition

$S = a + b + c$, in which S is the sum and a , b , and c are the addends.

1. $? = a + b + c$. 2. $S = ? + b + c$, or $S = a + ? + c$, or $S = a + b + ?$.

These formulas show that there are two general problems in addition: one direct and one indirect problem.

Subtraction

$Rem. = min. - sub.$, in which $Rem.$ is the remainder or difference, $min.$ is the minuend, and $sub.$ is the subtrahend.

1. $? = min. - sub.$ 2. $Rem. = ? - sub.$ 3. $Rem. = min. - ?$.

These formulas show that subtraction has three general problems: one direct and two indirect problems. Which are the indirect problems?

Multiplication

$Prod. = M \times m$ (also written Mm), in which $Prod.$ is the product, M is the multiplicand, and m is the multiplier.

1. $? = M \times m$. 2. $Prod. = ? \times m$. 3. $Prod. = M \times ?$.

How many general problems are there in multiplication?

Division

$$Q = \text{Div.} \div \text{div.} \text{ also written } Q = \frac{\text{Div.}}{\text{div.}}$$

In this formula Q is the quotient, Div. is the dividend, and div. is the divisor.

$$1. ? = \text{Div.} \div \text{div.} \quad 2. Q = ? \div \text{div.} \quad 3. Q = \text{Div.} \div ?$$

How many general problems are there in division?

The above formulas represent eleven types of problems which grow out of the four fundamental processes. These may be called fundamental problems because all the other arithmetic problems grow out of them.

Explain and solve each of these examples by stating (1) to which fundamental process the example belongs, (2) what is given, (3) what is to be found, (4) the value of the unknown term which is called x or some other letter.

Thus No. 5 is an example in subtraction. The remainder and the subtrahend are known. The minuend is x or the unknown number. The thinking is, "The minuend, or unknown number, is 75 more than 24." The unknown number is 99.

$$1. 3 + 7 + 8 = x$$

$$2. 31 = 4 + 9 + x$$

$$3. 84 = 100 - x$$

$$4. x = 74 - 39$$

$$5. 75 = x - 24$$

$$6. 68 \div 4 = x$$

$$7. x \div 5 = 20$$

$$8. 120 \div x = 24$$

$$9. 36 \times x = 144$$

$$10. 58 \times 3 = x$$

$$11. x \times 6 = 42$$

$$12. \frac{3}{4} \times x = 24$$

$$13. (3 + 8) - 6 = x$$

$$14. x - (56 + 20) = 24$$

15. $47 - a = 26$

16. $x + 3x = 8$

17. $\frac{1}{7}b = 3$

18. $\frac{2}{5}x = 8$

19. $x + \frac{2}{3}x = 10$

20. $3R^2 = 27$

21. $\$15 - \$x = \$9$

22. $\$36 = \$5 + \$b + \$24$

23. $z - 2 = 0$

24. $.3x = 9$

25. $.06x = 24$

26. $34 - x = 17$

27. $\frac{100}{2\frac{1}{2}} = y$

28. $80 - y = 16$

29. $z \div 4 = 17\frac{1}{2}$

30. $20 + x = 42$

31. $15 + 2x = 65$

32. $\frac{1}{2}x + \frac{1}{3}x = 10$

33. $\frac{48}{y} = .6$

34. $\frac{16}{x} = .08$

35. $\frac{8 \times 4}{c} = 64$

36. $3 \times t + 2 \times t = 10$

37. $8u + 4u = 48$

38. $15 \times a - 5 \times a = 20$

39. $\frac{1}{2}x - \frac{1}{3}x = 3$

40. $16 \div x = \frac{1}{2}$

41. $14 \div 2x = 7$

42. $100 \div 2\frac{1}{2}z = 20$

43. $16 \times 3 \times a = 96$

44. $12 \times 4 \times y = 144$

45. $D^2 \times 8 = 288$

46. $\sqrt{a} = 6$

47. $\sqrt{a} + 4 = 20$

48. $\sqrt{x} - 2 = 47$

49. $3x = 24 + 6$

50. $4a = 32 - 4$

51. $3t - 6 = 30$

52. $18c + 3c + 2c = 38$

53. $15t - 6t + 2t = 77$

54. $18 + a = 24$

55. $18 + 3a = 36$

56. $12 + 4a + 2a = 36$

57. $I = 640 \times .06 \times 2\frac{1}{2}$

58. $84 = 200 \times .06 \times T$

59. $h = \sqrt{6^2 + 8^2}$

60. $a = \sqrt{13^2 - 12^2}$

61. $169 = 5^2 + b^2$

62. $a^2 + 576 = 900$

63. $\pi R^2 = 78.54$

Expressing Problems as Equations

Translate the following problems into equations by observing this procedure.

1. Read your problem carefully.
2. Find out what is wanted.
3. Find out what is given.
4. From 2 and 3 determine the fundamental process to which the problem belongs.
5. Write the equation.

After you have written the equations for the entire exercise find the unknown number by the shortest method you know. Many of them you can think through without written interpretation.

1. The difference between two numbers is 18. The larger one is 48. Find the smaller.

THINKING.—We want to find the subtrahend, knowing the remainder and the minuend. This makes the problem one in subtraction.

STATEMENT.— $18 = 48 - x$.

2. A man earned \$ y in one day and \$6 more than this on the second day. In both days he earned \$14. How much did he earn the first day?

THINKING.—We want to find the earnings of the first day. We know the earnings of both days and how much more is earned the second day than on the first one. This makes the problem one in addition in which are given the sum of two addends and the second addend, to find the first addend.

EQUATION.— $y + y + \$6 = \$14$.

3. A general merchant had a tank holding $44\frac{1}{2}$ gallons of oil. One day he drew out $15\frac{3}{4}$ gallons, and the next day, $9\frac{1}{8}$ gallons. How many gallons were left in the tank?

4. 36 is 7 more than another number. Find the other number.

5. If 12 is subtracted from a , the result is 16. Find a .

6. After 16 is added to a number and then 14 is taken away, the result is 28. Find the original number.

7. After earning \$2.50 and spending \$1.10, I find that I have \$2.80. How much did I have before earning the \$2.50?

8. After having placed \$5 into my purse I found that it contained \$32. How much did the purse contain before the addition was made?

9. John has 5 marbles more than James and William has 3 more than John. They all have 43. How many does James have?

10. Robert has twice as many apples as Roy. Both have 24. How many does Roy have?

11. A newsboy earned $x¢$ on Monday. On Tuesday he doubled his Monday's earnings. On Wednesday he doubled Tuesday's earnings. His three days' earnings amounted to \$1.68. How much was earned on Monday?

12. The children of a school made small boxes to be filled with candy for a party. 600 were needed. Grades 3 to 7 made 20, 25, 83, 150, and 150 boxes. The 8th grade agreed to make the rest. How many did they agree to make?

13. Mary said to her classmates, "I am thinking of a certain number. If you add 6 to it and divide the sum by 3, your answer will be 24." What number did Mary have in mind?

14. Find the side of a square whose perimeter is 26.
15. A rectangle is twice as long as wide. Its perimeter is 60 in. Find the width.
16. Find p when $.6p = 180$.
17. 6% of s is 240. Find s .
HINT.—% means hundredths, then $.06s = 240$.
18. 15% of x is 225. Find x .
19. After 20% of c is added to c the sum is 12. Find c .
20. A 10% increase of a man's daily wages gave him \$4.40 per day. What were the wages before the increase?
21. After 40% of x was subtracted from x the remainder was found to be 36. Find x .
22. After reducing a certain debt 30% I find \$490 still unpaid. What was the original debt?

Using Equation Solutions Instead of Graph Solutions

Some time ago you solved the next 10 problems by means of graphs. Solve them now by translating each into an equation, letting the unknown number be represented by x or some other letter. After you have made the equation, find the value of the unknown number by thinking of your equation as an example in addition in which you need to find the addends.

1. James and John together have 8¢. James has three times as much as John. How much money has each?

Solution:

$$x¢ \text{ (John's money)} + 3x¢ \text{ (James' money)} = 8¢$$
$$4x¢ = 8¢$$

2. Ray and Robert together have 30 marbles. Ray has 4 times as many as Robert. How many has each?

3. Jane has twice as many apples as Mary and both have 15. How many has each?

4. Alice solved $\frac{2}{3}$ as many problems as Ada. Both solved 20. How many did each solve?

5. Two rectangles together have 192 sq. ft. If one is two times as large as the other, find the area of each.

6. A given rectangle is twice as long as it is wide. The perimeter is 48 inches. Find the length and the width.

7. In one school there are 7 more pupils than in a second school. In both there are 277 pupils. How many are there in each?

8. A farm yielded twice as many bushels of oats as wheat and four times as many bushels of corn as oats. The total yield was 1683 bu. What was the yield of each crop?

9. A lady paid twice as much for a hat and half as much for a pair of gloves as she paid for a pair of shoes. For all she paid \$21. How much did she pay for each?

10. Frank caught three fish whose total weight was 96 oz. The weight of the second was twice that of the first. The weight of the third was three times that of the first. Find the weight of each.

Solving Triangle Problems with the Equation

Problems similar to these you have solved hitherto by constructing a triangle and by reading the angles from your figure. You should now translate each of these problems into an equation and read the answer from the equation.

1. Two angles of a triangle are 40° and 50° . What is the third angle?

$x^\circ + 40^\circ + 50^\circ = 180^\circ$. Find x . Name the triangle.

2. Two angles of a triangle are equal. The third is 100° . What is the size of the equal angles? What kind of triangle is this?

3. Two angles of a triangle are equal, and their sum is equal to the third angle. Find the angles. What kind of triangle is this?

4. In a given triangle the largest angle is 3 times the smallest and the middle one is twice the smallest. Find the angles. Name the triangle.

HINT.—Call the smallest angle x° .

5. One acute angle of a right triangle is twice the other. How many degrees are there in each?

HINT.—You have here the sum of two addends, because you are dealing with only two angles.

6. Two angles of a triangle are in the ratio of 1 to 3, and the third angle is equal to their sum. Find the angles. Name the triangle.

HINT.—Call the smallest angle x° .

7. The largest angle of a triangle is 5 times the smallest. The middle angle is the difference between the largest and smallest. Find the angles.

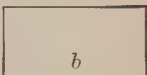
8. Angle A in a given triangle is $\frac{1}{5}$ of angle C and B is 3 times A . Find the angles. Name the triangle.

HINT.—Call the unknown number C .

9. In the triangle ABC the exterior angle at C is 90° . Angle A is $\frac{1}{3}$ of angle B . Find the angles. Name the triangle.

Formulas Used in Computing Lines and Surfaces

1. Rectangles.

(1) $A = b \times a$, also written ba , where a  b
 A = area, b = base, a = altitude. In the rectangle, base and altitude are length and width. See figure.

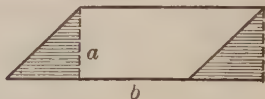
(2) Write the formula which tells how to find the area of a square.

(3) Write the formula for the perimeter of a rectangle.

2. Rhomboids.

$A = b \times a$, also written ba .

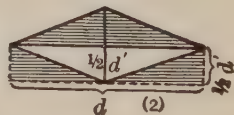
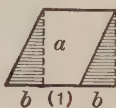
How would you find the base of a rhomboid, knowing its area and altitude?



3. Rhombuses.

(1) $A = b \times a$ or ba .

(2) $A = d \times \frac{d'}{2}$ where d



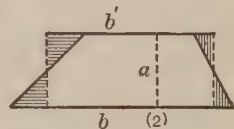
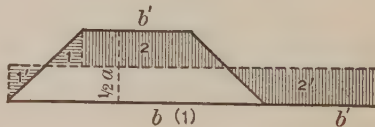
and d' are the diagonals.

4. Trapezoids.

(1) $A = (b + b') \times \frac{a}{2}$, where

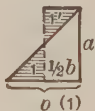
b and b' are the bases.

(2) $A = \frac{b + b'}{2} \times a$.

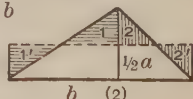
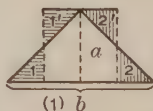
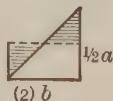


5. Triangles.

(1) $A = \frac{b}{2} \times a$.

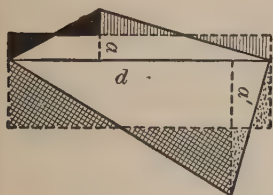


(2) $A = b \times \frac{a}{2}$.



(3) Write the formula for the perimeter of an equilateral triangle.

NOTE.—The figures show how the formulas for area are derived from the equivalent rectangle.



6. Trapeziums.

$A = d \times \frac{a + a'}{2}$ where d = a diagonal,
 a and a' are the altitudes drawn to
 this diagonal.

7. Circles.

C , D , and R represent circumference, diameter, and radius.

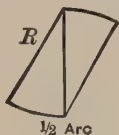
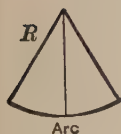
$$(1) C = \pi \times D = 2\pi R.$$

$$(2) A = \frac{C}{2} \times R.$$

$$(3) A = \pi R^2.$$

$$(4) A = \frac{\pi D^2}{4}.$$

$$(5) A = \frac{C^2}{4\pi}.$$



8. Sectors.

$$(1) A = \frac{\text{arc}}{2} \times R.$$

$$(2) A = \pi R^2 \times \frac{\text{central angle}}{360}.$$

$$(3) A = \pi R^2 \times \frac{\text{degrees in arc}}{360}.$$

9. Cones.

$$(1) \text{Convex surface} = s. h. (\text{slant height}) \times \frac{C}{2}.$$

$$(2) \text{Entire surface} = s. h. \times \frac{C}{2} (\text{convex sur-}$$

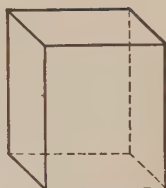
face) + πR^2 (area of base).



10. Prisms.

(1) Lateral surface $= P \times a$.

(2) Entire surface $= P \times a + \text{area of 2 bases.}$ $P = \text{perimeter of base.}$

**11. Cylinders.**

(1) Convex surface $= C \times a$.

(2) Entire surface $= C \times a$ (convex surface) $+ 2 \times \pi R^2$ (area of bases).

**12. Spheres.**

$$S = \pi D^2 = 4\pi R^2.$$

(1) $4\pi R^2$ means that the surface of a sphere is 4 times that of a circle whose radius is the radius of the sphere.

(2) What is the meaning of πD^2 ?

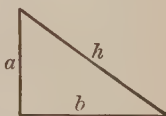
**13. Right Triangles.**

h , b , a , are hypotenuse, base and altitude. Tell the meaning of these formulas.

(1) $h = \sqrt{b^2 + a^2}$.

(2) $b = \sqrt{h^2 - a^2}$.

(3) $a = \sqrt{h^2 - b^2}$.



(4) Write the formula which shows how to find the altitude of an equilateral triangle when one side is given.

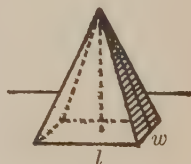
(5) Write the formula which shows how to find the diagonal of a square if one side is known.

14. Rectangular Pyramids.

(1) $P = 2(l + w)$.

(2) Lat. surface $= 2(l + w) \times \frac{s. h.}{2}$.

(3) Area of base $= l \times w$.



Formulas Used in Computing Volumes



1. Rectangular prisms.

$V = h \times l \times w$, where h , l , and w represent height, length, and width.

(1) What does the formula become when l and w are equal?

(2) Write the formula for the cube.

2. Cylinders.

$V = B \times a$, where B represents the area of the base, and a the altitude.

(1) Write the formula when the diameter of the base and the altitude are given.



(2) Write the formula when the radius of the base and the altitude are given.

(3) How would you find the altitude of a cylinder, knowing the volume and the radius?

3. Cones.

$$V = \frac{B \times a}{3}$$



(1) What is the difference between this formula and that of the cylinder?



(2) Construct out of heavy paper a cylinder and a cone with equal altitudes and diameters. Observe by actual measurement with the use of sand or sawdust that you can fill the cone three times with the contents of the cylinder.

4. Spheres.

$$V = \frac{\pi}{6} \times D^3$$

This formula says that the volume of a sphere is a little more than $\frac{1}{2}$ of the cube of the diameter.

Substituting Numbers for Letters in Formulas Finding the Value of Letters in Formulas

1. In the formula $P\Box = 2(a+b)$
 - (1) Find P when $a = 12$ and $b = 16$.
 - (2) Find a when $P = 48$ and $b = 8$.
 - (3) Find b when $P = 60$ and $a = 20$.
2. In the formula $A\Box = a \times b$
 - (1) Find A when $a = 12\frac{1}{2}$ and $b = 8$.
 - (2) Find b when $A = 240$ and $a = 15$.
3. In $A\Delta = \frac{a}{2} \times b$
 - (1) Find A when $a = 4\frac{1}{2}$ and $b = 12$.
 - (2) Find a when $A = 96$ and $b = 16$.
4. In $A_{Trapezoid} = \frac{b+b'}{2} \times a$
 - (1) Find A when $b = 24$, $b' = 18$, and $a = 12$.
 - (2) Find a when $b = 15$, $b' = 14$, and $A = 128$.
 - (3) Find b when $A = 36$, $a = 6$, and $b' = 4$.
 - (4) Find b' when $A = 96$, $a = 8$, and $b = 14$.
5. In $A_{Rhombus} = \frac{d \times d'}{2}$
 - (1) Find A when $d = 16$ and $d' = 12$.
 - (2) Find d when $A = 72$ and $d' = 16$.
 - (3) Find d' when $A = 72$ and $d = 18$.

HINT.—Numerically $\frac{d \times d'}{2} = \frac{d}{2} \times d' = d \times \frac{d'}{2}$.

How do the figures in (3) and (2) differ from each other?
How are they alike?

6. In $h^2 = a^2 + b^2$

- (1) Find h when $a = 12$ and $b = 5$.
- (2) Find a when $h = 10$ and $b = 6$.
- (3) Find b when $h = 26$ and $a = 24$.

7. In $h = \sqrt{a^2 + b^2}$

- (1) Find h when $a = 9$ and $b = 40$.
- (2) Find a when $h = 25$ and $b = 24$.
- (3) Find b when $h = 61$ and $a = 11$.

8. In $s. h. cone = \sqrt{a^2 + R^2}$

- (1) Find $s. h.$ when $a = 16$, $R = 12$.
- (2) Find a when $s. h. = 39$, $R = 15$.
- (3) Find R when $s. h. = 17$, $a = 15$.

9. In $V_{cone} = \frac{\pi R^2 a}{3}$

HINT.—Numerically $\frac{\pi R^2 a}{3} = \pi R^2 \times \frac{a}{3} = \frac{\pi R^2}{3} \times a = \frac{\pi a}{3} \times R^2$.

- (1) Find V when $R = 8$ and $a = 15$.
- (2) Find a when $V = \frac{2200}{7}$ and $R = 5$.
- (3) Find R when $V = 96\pi$ and $a = 8$.

10. If you double each of the sides of a square, what happens to the perimeter? What happens to the area?

Solving Mensuration Problems with Formulas

All terms in a mensuration formula are abstract. Therefore, in solving problems with such formulas all parts of the solution which involve a formula should be kept abstract as shown in the next solutions.

1. Find the area of a circle whose radius is 8 ft.

FORMULA.— $A = \pi R^2$.

SOLUTION.— $A = \frac{22}{7} \times 8 \times 8 \times 1$ sq. ft.

The area = $201\frac{1}{7}$ sq. ft.

2. A lawn in the shape of a rhomboid is 50 feet long and 36 feet wide. How much will be the cost to sod it at 25¢ a square yard?

The formula employed in the solution is $A = b \times a$.

SOLUTION.—Cost or $\$x = \$.25 \times \frac{50 \times 36}{9}$.

$\$x = \$50$.

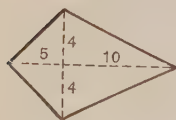
3. There are 40 acres in a rectangular field which is 40 rods wide. How long is it?

4. Find the area of a triangle whose base is 24 feet and whose altitude is 16 feet.

5. A triangle whose base is 25 feet contains 200 square feet. Find the altitude.

6. The lower base of a trapezoid is 20 feet; the upper is 16 feet; the altitude is 8 feet. Find the area.

7. A kite is in the form of a trapezium whose diagonals intersect each other at right angles as shown in the figure. Find the area and the perimeter.



8. If the diameter of the moon is 2000 miles, what is the area of the portion visible to us at full moon?

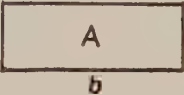
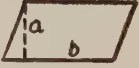
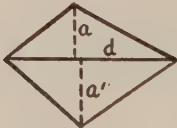
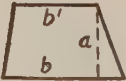
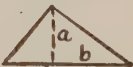
9. Find the area of the earth, calling it a sphere whose diameter is 8000 miles. (Call π 3.1416.)

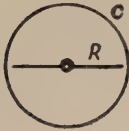
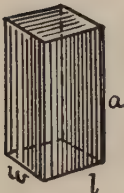
On a sheet write the numbers in order from 1 to 24. Read carefully each of the next equations. Place a check on your sheet after those numbers which enumerate correct equations. Change those equations which you think are false to true ones. Write only the corrected equation after its proper number. For example, after No. 12 on your paper you should write $C = \pi D$ because No. 12 is a false equation as it stands in this exercise, but after No. 1 on your paper you should place a check because it is a true equation.

1. *Dividend* $\div$ *quotient* = *divisor*.
2. *Divisor* $\times$ *quotient* = *dividend*.
3. *Multiplier* $\times$ *product* = *multiplicand*.
4. *Minuend* + *subtrahend* = *remainder*.
5. *Divisor* $\times$ *quotient* + *remainder* = *dividend*.
6. *Commission* $\div$ *amount of sale* = *rate of commission*.
7. *Cost* $\div$ *profit* = *rate of profit*.
8. $A \square$ = *the square of one-half of the diagonal*.
9. $S.p. - c = l$.
10. $l = c - s.p.$
11. $I = P \times R \times T$.
12. $C \odot = \pi R$.
13. $A \odot = C \times R$.
14. $A \square = s^2$.
15. $A \triangle = a \times b$.
16. $C \odot = D \div \pi$.
17. $V_{sph.} = \frac{1}{6} \pi D^2$.
18. $A \square = a \times b$.
19. $S_{cu.} = 4a^2$.
20. $V_{cu.} = a^3$.
21. $P \square = 4a^2$.
22. $V_{cyl.} = \pi R \times a$.
23. *Convex Surface* $_{cone} = \frac{s.h. \times \pi R}{2}$.
24. *Convex surface* $_{cyl.} = \pi R^2 \times a$.

A Matching Test With Mensuration Formulas

Rearrange the formulas so that they will correspond with the names and figures found in the first and second columns. If you can not find a figure or formula for any name in the first column, supply it.

Name	Figure	Formula
1. Rectangle		$A = b \times a.$ $A = \text{area}.$ $P = 2a + 2b = 2(a + b).$ $P = \text{perimeter}.$
2. Square		$A = b \times a \text{ or } ba.$ $A = d \times \frac{d'}{2} = \frac{d d'}{2}.$
3. Rhomboid		$A = d \times \frac{a + a'}{2}.$
4. Rhombus		$C = \pi D = 2\pi R.$ $A = \frac{C}{2} \times R.$ $A = \pi R^2.$
5. Trapezium		$A = \frac{\pi D^2}{4}.$ $A = \frac{C^2}{4\pi}.$ $A = b \times a.$
6. Trapezoid		$A = \frac{\text{arc}}{2} \times R.$
7. Triangle		$A = \pi R^2 \times \frac{\text{central angle}}{360^\circ}.$ $A = \pi R^2 \times \frac{\text{degrees in arc}}{360^\circ}.$

Name	Figure	Formula
8. Circle		$A = (b + b') \times \frac{a}{2}.$ $A = \frac{b + b'}{2} \times a.$
9. Sector		$A = a \times \frac{b}{2}.$ $A = \frac{a}{2} \times b.$
10. Prism		$A = \frac{a \times b}{2} = \frac{1}{2}ab.$ $A = a \times a = a^2.$ $A = d \times \frac{d}{2} = \frac{d^2}{2}.$
11. Cylinder		$P = 4a.$ $\text{Surface} = 4\pi R^2 = \pi D^2.$ $V = \frac{\pi D^3}{6} = \frac{\pi}{6} \times D^3.$ $V = \text{Volume}.$
12. Cone		$P = 2(l + w).$ $\text{Lateral surface} = 2(l + w) \times a.$
13. Sphere		$\text{Entire surface} = 2(l + w) \times a + 2lw.$ $V = l \times w \times a = lwa.$

Read the problem carefully; then write the statement or tell the answer with the help of the formulas which you studied in this chapter.

1. How many bushels of wheat in a rectangular bin 12 ft. long, 8 ft. wide, and 7 ft. high if 1 cu. ft. = .8 bushel?

2. A boy built a coal bin 15 ft. long, 10 ft. wide, and 6 ft. high. How many tons of soft coal will it hold if it can be filled to within $\frac{1}{2}$ foot of the top?

3. John saw a cylindrical silo whose height, he thought, was 36 ft. and whose diameter was 15 ft. Find the volume based on his estimates.

4. How much air space in a conical tent whose diameter is 18 ft. and whose height is 12 ft.?

5. This battery jar is 6 inches in diameter and 8 inches high. Dick thought it would hold a gallon. Was he too high or too low? How much?



6. The dimensions of these three bodies are equal. Their volumes are as 1:2:3. What does this mean?



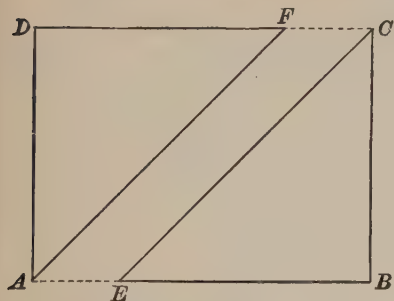
7. Jennie said, "I can place the sphere into the cylinder and then add one cone full of water." Was she right?

8. How many strokes must be made by a pump, having a cylinder 16 in. long and 4 in. in diameter, to throw a barrel of water ($31\frac{1}{2}$ gal.) if each stroke empties the full volume of the cylinder?

9. The rate of flow out of a two-inch artesian well is one foot per second. What is the flow in gallons in 24 hours if $7\frac{1}{2}$ gal. = 1 cu. ft.?

10. Find the convex surface of a cone whose altitude is 10 ft. and the diameter of whose base is 12 ft.

11. Find the area of a sector whose radius is 10 inches and whose arc is 10 inches.



12. The accompanying figure represents a garden in the form of a rectangle, 40 ft. long and 30 ft. wide, with a walk extending across it. The two gates, located at FC and AE respectively, are 6 ft. wide.

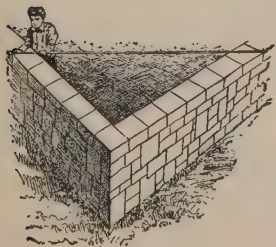
(1) Find the length CE ; also the perimeter of the walk.

(2) What is the area of the walk?

(3) What % of the entire figure is the area of the walk?

13. One side of an equilateral triangle is 12 ft. Find the altitude. Find the area.

14. The base of an isosceles triangle is 18, one of the equal sides is 15. Find the altitude and the area.



15. A contractor wished to know if the foundation of a building was square. He took a point 6 ft. from the corner on one wall and a point 8 ft. from the same corner on the other wall. What must have been the distance between these points if the foundation was square?

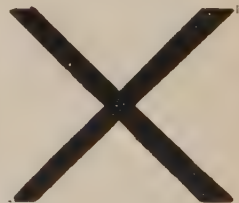
16. If a cubic foot of ice weighs $57\frac{1}{2}$ lb., how many tons of two-foot ice are there on a circular pond whose diameter is 400 ft.?

17. The inside diameter of a line of fire hose is 4 in. How many gallons of water will flow through this line in an hour if the water moves at the rate of 30 ft. a minute?

18. A gallon of paint sometimes covers 250 sq. ft. At this rate what will the paint, necessary to put two coats on the convex surface of a cylindrical silo 28 ft. high and 14 ft. in diameter, cost at \$2 per gallon?

19. If the diameter of a conical tent is 18 ft. and the slant height is 15 ft., find the distance from the center of the base to the apex.

20. The letter X on this page consists of two rhomboids with the following dimensions: length, long side, $1\frac{1}{2}$ in.; short side, $\frac{3}{16}$ in.; altitude, $\frac{1}{8}$ in. The letter is cut from a rectangle $1\frac{1}{8}$ in. $\times$ 1 in.



A boy, using this letter as a working model, made another one whose dimensions were 4 times as large for a Christmas design.

- What were the dimensions of his letter?
- If he made his letter in one piece, what were the minimum dimensions of the cardboard on which he laid it off?
- What % of the cardboard was cut away?
- How could you make the letter with less cardboard?
- He covered the surface of the design with gold cloth. How much did he need? He edged the design with silver tinsel. How many inches did he use?



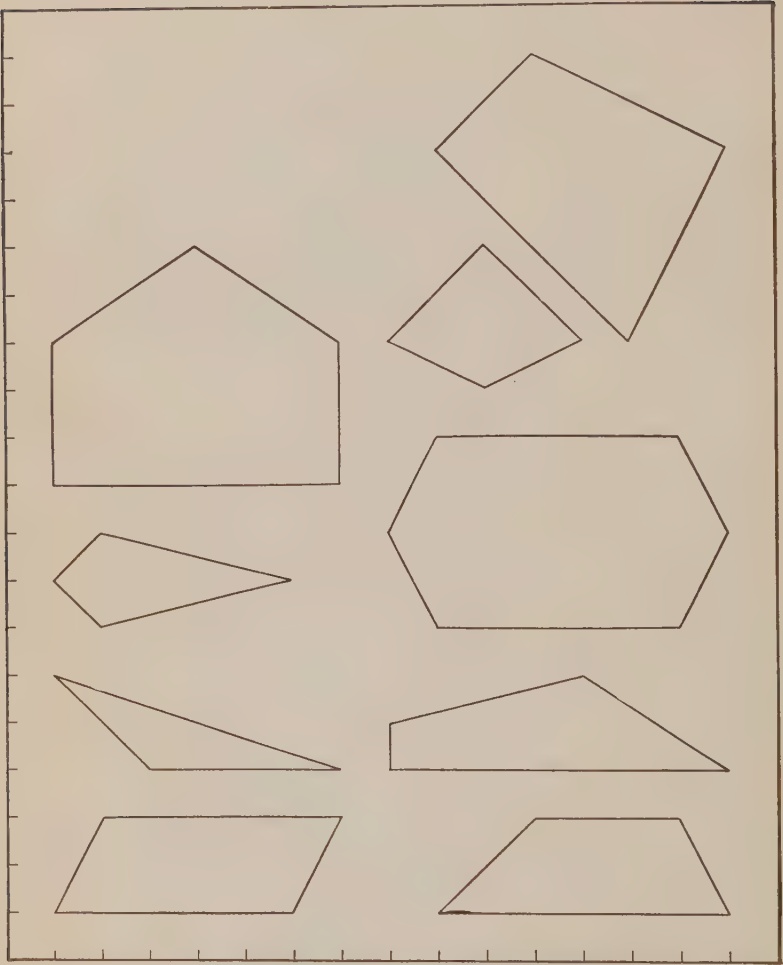
Silo Problems

1. A farmer built a cylindrical silo 10 feet in diameter and 20 feet high, inside measurements. If a cubic foot of silage (green corn fodder) weighs 36 pounds, how many tons are there in the silo when it is $\frac{3}{4}$ full?

2. A silo 32 ft. high and 16 ft. in diameter is full of silage. If 50 cu. ft. of silage weigh a ton, and a cow eats 30 lb. of it in a day, how long will this silage last 50 cows?

3. To keep silage in good condition a layer 2 inches deep should be used daily. A silo is 12 feet in diameter and is filled to such a depth that a cubic foot of silage weighs 35 pounds. How many cows should be kept to feed a layer 2 inches deep every day if each cow needs 34 pounds daily? Call $\pi \frac{22}{7}$.

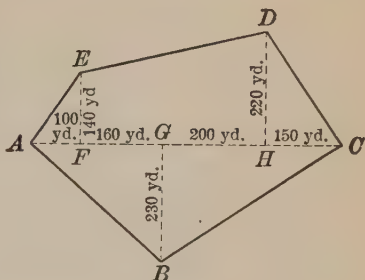
4. The farmer who owns the silo in the previous problem wishes to keep enough cows to lower the silage in the silo 3 inches daily. How many must he keep?



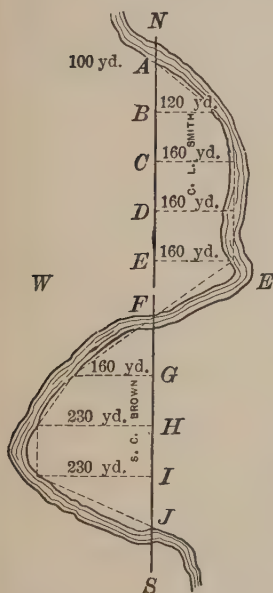
Compute the area of each of the figures within this rectangle. What % of the large rectangle is the sum of the areas of the figures within it?

1. Find the area of a field in the shape of a polygon $ABCDE$, if $AC=610$ yd.; $EF=140$ yd.; $HD=220$ yd.; $BG=230$ yd.; $AF=100$ yd.; $AG=260$ yd.; and $AH=460$ yd.

2. The two parallel sides of a field are 120 rd. and 84 rd., and its width is 40 rd. Find the area in acres.



3. A line NS , running north and south, is the boundary line between the farms of S. C. Brown and C. L. Smith. In order that each party may have all of his land on one side of the stream, it was agreed that Brown should sell to Smith all the land Brown owned inside of the south bend of the river and Smith should sell to Brown all the land Smith owned inside of the north bend of the river, the price to be \$100.00 per acre in each case. The surveyor set stakes at $A, B, C, D, E, F, G, H, I$, and J , 100 yd. apart, and measured the distance from each point to the river on a line at right angles to NS , as indicated in the diagram. It was agreed that the oblique side of each triangle or polygon so made should be considered a straight line from point to point. Which party is to make a cash payment in the transfer and how much?



It was agreed that the oblique side of each triangle or polygon so made should be considered a straight line from point to point. Which party is to make a cash payment in the transfer and how much?



A MODERN FILTERING PLANT AND SETTLING BASIN

In some villages each residence has its own well or cistern; in others there is a well at the public square or elsewhere for all the people; in others there is a large reservoir from which the water is piped to each home or business place. The last method is the one used by villages and cities with modern conveniences.

1. What is meant by modern conveniences?
2. State several reasons why the last method is most desirable?

Some cities are so situated that they must draw their water supply from a nearby stream or lake. This requires that the impurities be removed by settling or filtering, or by both, before the water is fit for use.

Name several impurities that should be removed.

1. A certain village in Ohio has in round numbers a population of 500. The reservoir for the town supply is a cylindrical tank 36 ft. long with a 9-foot diameter. What is the capacity of the reservoir? Count $7\frac{1}{2}$ gal. to the cu. ft.

2. The pump which supplies the tank in the first problem throws 2.7 gal. to the stroke and makes 23 strokes to the minute. How long, with steady pumping, will it require to fill the tank?

3. If the pump is driven on an average of 8 hours daily, how many gallons of water are allowed daily to each inhabitant in the village of problem one?

4. In a city whose population is 750,000, the cost of purifying its water in 1918 was \$7.37 per million gallons. How much was that per gallon? At this rate how many gallons were purified for one cent?

5. In the same city the average daily consumption for 1917 was 104.3 million gallons. How much was that daily per person? How much was the total consumption for 1917? Express your answer in millions.

6. Using the data in 4 and 5, determine the average cost of purifying the water used by each person in a year.

7. St. Louis yearly purifies 36 billion gallons of water by adding 5.35 grains of lime, .68 grains of iron sulphate (copperas), and .58 grains aluminum sulphate (alum) to each gallon. How many tons of each of these materials are used yearly?

CHAPTER VIII

METRIC MEASUREMENTS

The metric system was originated in France at the time of The French Revolution, about 1793. It now is the standard system of weights and measures for all the important civilized countries in the world except the United States and Great Britain including Canada. It was legalized in this country by an Act of Congress in 1866. The general use of metric weights and measures in this country has been advocated by many important organizations of scientific men, of manufacturers, and of importers and exporters. At the present time the system is used in scientific text books, in physical and chemical laboratories, and in some departments of our national government. Because of the general use of the metric system throughout the world and because of its growing importance in our own country, you should have a reading knowledge of its weights and measures.

The fundamental unit of the metric system is the meter because from it, or its sub-divisions, is derived the standard unit for each of the other tables. The meter was arbitrarily taken as the length of the one ten millionth part of the quadrant of the meridian running through Paris. For practical purposes you may call the meter 40 inches; a more accurate value is 39.37 inches.

The Metric Tables

The important metric tables with which you should have an acquaintance are those dealing with measures of

length, of surface, of contents, of capacity, and of weight. The standard units for these measures are the meter for length, the square meter for surface, the cubic meter or cubic centimeter for contents, the liter (pronounced *lêter*) for capacity, and the gram for weight.

By remembering that the meter is practically 40 inches, that the liter is a cubic decimeter (64 cu. in.), and that the gram is the weight of a cubic centimeter (.064 cu. in.) of distilled water at its maximum density (39.2° Fahr.), the metric measures of the above named tables may be easily converted into the approximate equivalents of our tables.

Measures of Length

The standard unit of measures of length is the meter (m.) whose length is 39.37 in. Note in the table how the names of the other measures are derived.

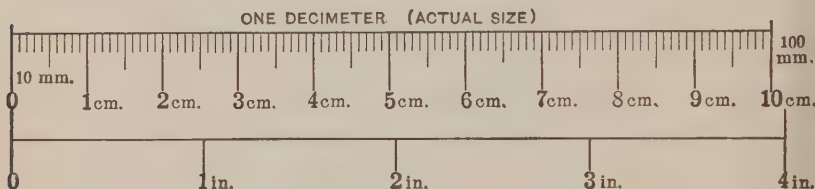
TABLE

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.), 39.37 inches
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 kilometer (Km.), .62 mi.

Practical Equivalents

1 meter	= 40 in.
1 decimeter	= 4 in.
1 dekameter	= 2 rd.
1 kilometer	= $\frac{5}{8}$ mi.

See the figure below for the exact and approximate equivalents of the decimeter and the centimeter.



Meaning of the Prefixes in the Meter Table

Milli means one-thousandth.

Deka means ten.

Centi means one-hundredth.

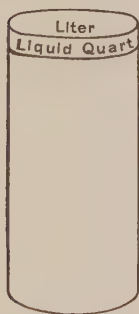
Hekto means one hundred.

Deci means one-tenth.

Kilo means one thousand.

These prefixes and their meanings should be learned because they are used with liter and with gram in the tables of capacity and of weight.

Our words *mill*, *cent*, and *dime* are modifications of the above prefixes and mean a thousandth of a dollar, a hundredth of a dollar, and a tenth of a dollar.



Comparison of the liter and liquid quart, reduced to one fifth of actual size.

Measures of Capacity

The liter is the standard unit of this table. It is the volume of a cube whose edge is one decimeter.

TABLE

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 liter (l.)
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 hektoliter (Hl.)
10 hektoliters	= 1 kiloliter (Kl.)

Practical Equivalents

1 liter = 64 cu. in.	1 hektoliter = 2.8 bu.
1 liter = 1.1 liquid qt.	1 dekaliter = $1\frac{1}{8}$ pk.
1 liter = .9 dry qt.	1 centiliter = .88 gi.

The above table combines the English liquid and dry measure tables. Therefore, the hektoliter is used to measure grains, vegetables, fruits in large quantities; the liter is used to measure small fruits and liquids in small quantities. The milliliter is used to measure liquid medicines in very small quantities.

Measures of Weight

The gram is the standard unit of this table. It is the weight of a cube of distilled water at 39.2° Fahr. whose edge is one centimeter. The gram weighs 15.3 grains. It is used in weighing small quantities.

The kilogram, also called kilo, is the weight used in ordinary business. It is the practical equivalent of 2.2 lb.

The metric ton (2204.6 lb.) is used in weighing heavy articles. It is the weight of a cube of water whose edge is one meter.

TABLE

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 <i>gram</i> (g.) 15.3 grains
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 kilogram (Kg.)
1000 kilograms	= 1 metric ton (M.T.)

Practical Equivalents

- 1 gram = 15 grains or .035 avoirdupois oz.
 1 kilogram = 2.2 lb.
 1 metric ton = 1.1 tons



Comparison of two-pound and one-kilogram brass weights, reduced one-half

Measures of Surface

The square meter, called centar, may be regarded as the standard unit in this table. The square meter is used to measure such surfaces as would be measured by the square yard in our system.

The square dekameter, called ar, is used to measure land such as farms and fields. Larger areas are expressed in square hektometers, called hektars, or in square kilometers.

TABLE

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.)
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 square meter (sq. m.)
100 square meters (centars)	= 1 square dekameter (sq. Dm.)
100 square dekameters (ars)	= 1 square hektometer (sq. Hm.)
100 square hektometers	= 1 square kilometer (sq. Km.)

Practical Equivalents

- 1 square meter (sq. m.), or centar = 1.2 sq. yd.
 1 square dekameter (sq. Dm.), or ar = almost 4 sq. rd.
 1 square hektometer (sq. Hm.), or hektar = 2.5 acres.
 1 square kilometer (sq. Km.) = .4 sq. mi.

The cubic centimeter is used in measuring small volumes. The cubic meter is used where in our system ordinarily the cubic yard, or the cord, would be used.

TABLE

1000 cubic millimeters (cu. mm.) = 1 cubic centimeter (cu. cm.)

1000 cubic centimeters = 1 cubic decimeter (cu. dm.)

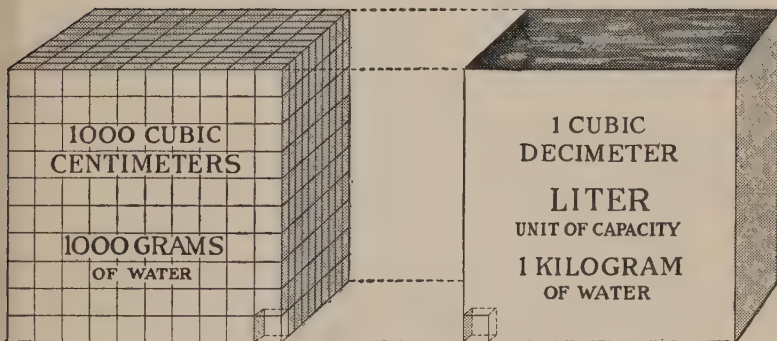
1000 cubic decimeters = 1 cubic meter (cu. m.)*

*The cubic meter when used in measuring wood is called a *stere*.

Practical Equivalents

1 cubic decimeter = 64 cu. in.

1 cubic meter = 1.3 cu. yd.



The figure shows the relation of the centimeter to the decimeter, of the square centimeter to the square decimeter, of the cubic centimeter to the cubic decimeter, of the cubic centimeter to a gram of water, and of the cubic decimeter to the liter and to a kilogram of water.

By means of this figure, volume in cubic units is easily converted into measures of capacity and, with the help of a table of densities (specific gravity), to measures of weight.

Changing Metric Measures to Larger or Smaller Units

Read the missing numbers by shifting the decimal point and supplying zero when necessary.

1. 804 mm, = ____ cm. = ____ dm. = ____ m.
2. .375 m. = ____ dm. = ____ cm. = ____ mm.
3. 3 m. 2 dm. = ____ dm. = ____ cm. = ____ mm.
4. 3 dm. 8 cm. = ____ dm. = ____ cm. = ____ mm.
5. 85 dm. = ____ m. = ____ Dm.
6. 500 g. = ____ Dg. = ____ Hg. = ____ Kg.
7. .25 g. = ____ dg. = ____ cg. = ____ mg.
8. .85 Hl. = ____ Kl. = ____ Dl. = ____ l.
9. 759 ml. = ____ cl. = ____ dl. = ____ l.
10. 6 m. = about ____ in. = about ____ ft.
11. 10 g. = about ____ gr. = about ____ oz.
12. 20 l. = about ____ liquid qt. = about ____ dry qt.

Using Metric Units in Reading about The Great War

The following expressions are from letters and conversations by our soldiers who fought in France in The Great War. To understand what they said you need to know the meaning of the metric unit of measure in terms of our measure.

1. "At Ypres our front line trenches were not more than 25 meters from those of the enemy." How many feet was that?

2. "In the Champagne our front line was 100 meters from that of the enemy." How many feet?

3. "From Nancy to Metz is but 35 kilometers; to Strasbourg only 170 kilometers." How many miles is each city from Nancy?

4. "In the trenches about Ypres a pump was installed at every 100 meters distance." How many feet were the pumps apart?

5. "German shells scarcely ever reached more than 22 kilometers to the rear of our firing line." How many miles was that?

6. "Small parts of the walls of Ypres were left standing to the height of 3 meters." How high is that in yards?

7. "Our front line trench at 25 meters was within talking distance of Fritz and his Mauser rifle." How many feet away was the German front line trench?

8. "I saw an interesting air duel one day directly overhead at an altitude of 1500 meters, I would judge." How many yards was that?

9. "The communicating trench was usually dug 90 centimeters wide and 1 meter 50 centimeters deep. The evacuation trench (passage way for wounded soldiers) was 1 meter 20 centimeters wide." What are the dimensions of these trenches in feet and inches?

10. "You-Yous are rifle grenades with a copper stem about 25 centimeters long." How many inches is that?

11. "The French 75's (75 millimeter guns) were the most accurate of any artillery on the front. The 88's of the Austrians were probably the next most accurate." What are 75 and 88 millimeter guns?

Practical Problems with Metric Measures

1. Make a meter stick for your class room. Lay off on it decimeters and centimeters.

HINT.—40 inches is an equivalent close enough for ordinary purposes.

2. Try to make a step one meter long. Measure the step made. Compute your % of accuracy.

3. Estimate, then measure, the length and width of the top of your desk, both in inches and in centimeters.

HINT.—You should be provided with a ruler showing both English and metric units. If the school does not furnish such rulers, make one out of heavy white paper, using the figure on a previous page as a guide.

4. Find the difference in centimeters between the meter stick and the yard stick.

5. Find the difference in millimeters between the foot rule and one fourth of the meter stick.

6. Find the perimeter of a French schoolroom 9.2 m. long and 7.2 m. wide. Compare this with the perimeter of your schoolroom.

7. Draw the yard and the meter to scale representing 8 in. with one inch.

8. A little French boy to reach school had to walk 60 meters due east, then 50 meters due north, and then 60 meters due west. Draw his path to scale. How far and in what direction was his stopping point from his starting point?

9. A certain French soldier was 1.6 m. tall. His cousin, an American soldier, was 5 ft. 10 in. Show the height of these soldiers by a convenient scale.

CHAPTER IX

TRADE AND TRANSPORTATION

The farmer cannot consume all the wheat and meat he can produce, neither can he produce profitably all the clothing he needs. The coal miner needs only a small portion of the coal he digs. He does not grow any of the wheat which he needs for bread.

Among civilized people it is a common thing for men to produce more of one commodity than they can consume and to need many things which they cannot produce. Meeting these needs of people and disposing of the surplus of the producer give rise to trade and transportation.

The business man buys the surplus and often sends it to another man whose business it is to get this surplus into the hands of those who need it.

1. Do you know any producers in your community who sell directly to consumers?

2. Do you know any business men who buy from producers and sell to consumers?

3. Do you know any business men who buy from other business men and sell to consumers?

4. Why must the consumer pay more to the business man than to the producer?

5. Write and solve a problem about a business man and a consumer in your community.

How Business Is Done

Before computing his profit the merchant adds to the cost of the goods the **expense of doing business**. This expense varies from 10% to $33\frac{1}{3}\%$ of the cost. Such items of expense as rent, tax, clerk hire, advertising, etc., are called **overhead charge**, or **overhead**.

Profit is the difference between what is actually received for an article and its **total cost**, also called **gross cost**.
Total cost = cost + transportation (freight, drayage, etc.) + overhead. The last two addends are called the expense of doing business, or expense.

Business men often compute their rate of profit on the selling price.

Problems about Business

1. A merchant bought goods for \$1800 and sold them for \$2500. Find his profit if the expense of doing business was 25% of the cost. What was his rate of profit on the selling price?

2. A merchant bought men's shirts at \$15 per dozen. His selling expense including overhead was $33\frac{1}{3}\%$ of the cost. He marked the shirts to sell at \$2.50 each, but due to a slow season he actually received only an average price of \$2.25. Find his rate of profit on the selling price.

3. A manufacturer sells his product at \$10 per bbl. The cost of materials and of manufacturing amounts to \$8 per bbl. The overhead is $12\frac{1}{2}\%$ of the selling price. Find the rate of profit on the total cost.*

* Rate of profit may be computed on the net cost, on the total cost, or on the selling price. Among business men there is no uniform practice on this point.

4. Construct and solve a problem about a grocery business in your community, using cost, overhead, selling price, and profit.

5. A dealer bought 30 bu. of apples at \$1.25 per bu. and sold them at 40¢ a peck. At the same time he purchased 100 lb. of English walnuts at 15¢ a lb., which he retailed at 24¢ a lb. Which purchase was the more profitable in money and how much? Which yielded the greater rate of profit on the selling price?

6. A grocer buys peaches at \$2.00 per dozen cans and sells them at 25¢ per can. It costs him 2¢ a can to sell them. Find the gross cost. What is his rate of profit on the selling price?

7. An Oklahoma grower of cantaloupes received \$1.20 per crate. The crating cost 10¢ per crate; the express from Oklahoma to Denver, Colo., was 35¢ per crate. The consumer at Boulder (near Denver) paid \$3.50 per crate for his cantaloupes. Who got the difference between \$3.50 and the other items named above? How much is this difference? What % is it of what the farmer received?

8. An Ohio farmer in 1918 had 100 hens which laid during the year 809 dozen of eggs. These eggs were sold at an average of 43¢ per doz. He sold during the year poultry for which he received \$38.70. What was the gross income per hen? How would you find the net income?

9. A broker bought for his firm goods for which he paid \$1200. He received 3% brokerage. Freight and other delivery charges amounted to \$20. Advertising cost \$10. Find the selling price to gain 30% on the entire cost.

How Transportation Arises

Much trade and transportation arise also in the process of converting raw materials into the finished products and in getting these products into the consumer's hands. Cotton grown in the field is taken to the gin (what is done to it here?), then to the compress (what happens here?), then it is sent to the cotton mill (why?), then some of it goes to the shirt factory, and finally some of these shirts may be found in the home of the man who grew the cotton.

To get commodities from a region which can very abundantly produce them, to a region whose people want to buy them, because they can do other work with greater profit, requires some system of transportation, such as, good roads, railroads, or steamship lines.

Good Roads

Good roads may be built of dirt, gravel, macadam (crushed rock), brick, or concrete. Some good roads are



Fig. I Loose Gravel



Fig. II Earth with Dust Top



Fig. III Packed Gravel



Fig. IV Waterbound Macadam

Using Concrete

1. Concrete is a mixture of cement and sand; or of cement, sand, and gravel or crushed stone.

2. It is used in the construction of walks, roads, foundations, floors, walls, curbs, fence posts, trolley posts, water troughs, etc. How else have you seen concrete used?

3. Cement is bought by the sack, or by the barrel. A sack contains 1 cu. ft., and a barrel contains 4 cu. ft.

4. Sand, gravel, and crushed stone are bought by the cubic yard in many places. In large cities sand and gravel are commonly sold by the ton; crushed stone by the hundred cubic feet. A cubic yard of sand or gravel weighs approximately 2700 lb.

5. Since concrete has various uses, the materials which go into it are mixed in different ratios. A rather common mixture for outside work is 1:2:3 (read one two three).

Concrete Table

Proportion of Volume of Material Used			Amount of Material in 1 cu. yd. of Concrete		
Cement	Sand	Stone or Gravel	Cement	Sand	Stone or Gravel
1	1		bbbl. cu. yd. 4.75 = .7	cu. yd. .7	
1	2		3.25 = .48	.96	
1	2	3	1.75 = ?	?	?
1	2	4	1.5 = ?	?	?
1	3	6	1. = ?	?	?

This means that for every cubic yard of concrete to be used a certain amount of cement (say 1.75 bbl.), 2 times as much sand, and 3 times as much gravel or stone are required. For foundations a 1:3:6 mixture may be used. What does this mean?

6. A cubic yard of space requires about 1.45 cu. yd. of material. In actual practice an additional 10% is allowed.

1. Find the value of all the question marks in the table.
2. Using the values found in the table, determine the amount of each material required for 12 cu. yd. of concrete of a 1:2:3 mixture.*
3. How much of each material will be required for a top coat containing 8 cu. yd. of concrete of a 1:1 mixture?
4. How many cubic yards of materials will be required for a concrete base 6 inches thick in a basement 36 ft. $\times$ 25 ft.?

HINT.—Use a 1:2:3 mixture.

5. A walk 60 ft. long and 3 ft. wide is to be built with a concrete base (bottom) 5 inches thick of a 1:2:3 mixture, and a top coat 2 in. thick of a 1:1 mixture. How much of each kind of material is required if 10% is allowed for waste? Find the cost of the materials needed for this walk, using current prices.

6. If cement costs 75¢ a sack, sand \$2.50 a cu. yd., and crushed stone \$2.16 a cu. yd., find the cost of the materials in a 1:2:3 mixture sufficient to make 200 cu. yd. of concrete.

7. The foundation wall for a building 120 ft. long and 80 ft. wide, outside measurements, is to be 3 ft. wide and 9 ft. high. The concrete is to be a 1:3:6 mixture. Find the cost of the materials if the cement is worth 75¢ a sack, and if the sand and gravel are each worth \$2.70 a load (1 cu. yd.), (1919 St. Louis price). Add 10% to each kind of material for waste.

* NOTE.—Any part of a sack or bbl. of cement, or of a cu. yd. of sand or stone, should be considered a full sack, bbl., or cu. yd.

Railroads

In 1919 there were in this country 264,000 miles of railroad, representing an investment of over \$19,000,000,000. Even this large mileage was not enough to meet actual needs.

1. What was the average investment per mile?

2. It is 3190 miles from New York to San Francisco. How many tracks between these cities would our railroad mileage make?

3. The total length of the railroads of the world in 1919 was 730,900 mi. The United States had 264,000 mi. What % of the world's railway mileage was that of the United States?

Methods of Shipping Goods

Merchandise may be shipped by freight, by express, or by parcel post.

1. When is shipment by parcel post preferable? Do you know the maximum weight which can be shipped by parcel post? If not, see page 308.

2. Would you ship watermelons from Texas to Chicago by freight or by express? Why?

3. Would you ship coal from the Illinois mines to Chicago by freight or by express? Why?

4. The freight charge on a car of tomatoes containing 500 crates, shipped from Miami, Fla., to Chicago, was \$237.56. The cost of refrigeration was \$80 and the drayage charge was 3¢ per crate. Find the total transportation cost per crate

5. Find the freight charge at 45¢ per 100 lb. on a car load of books weighing 39,200 lb., shipped from Chicago, Ill., to Louisville, Ky.

6. What was the freight rate per 100 lb. when \$14.04 was paid on a shipment weighing 520 lb.?

7. The freight rate, first class, from Chicago, Ill., to Oklahoma City, Okla., was \$1.87½. Find the freight on a shipment of 3520 lb.

8. What is the total freight on four shipments of books whose weights and rate per 100 lb. are as follows: 220 lb. @ \$3.31½, 200 lb. @ \$2.67½, 340 lb. @ 72¢, 1060 lb. @ \$1.19½?

9. In 1919 a rate of \$35 per ton of 2240 lb. was made for shipping wheat from Buenos Aires to New York City. How much was that per bushel?

10. In 1919 the Argentine Government fixed the minimum price for export wheat at \$1.55 per bushel. At this rate, what did a bushel of Argentine wheat cost in New York if there was no import duty?

11. Can the farmers of our country compete with Argentine wheat growers without an import duty, as long as Argentine maintains an export price of \$1.55 per bushel, and the shipping charge is \$35 per ton of 2240 lb.?

12. The freight rate on wheat from Buenos Aires to Liverpool, in 1919, was 34¢ per bushel. The British Government in the same year fixed the sale price of imported wheat at \$2.40 per bushel. Which was more profitable to the Argentine exporter, to have sold his wheat in New York at \$2.48, or in Liverpool at \$2.40? How much?

13. Does the American wheat grower profit by the conditions described in problems 10, 11, and 12? Why?

14. A Chicago firm had a shipment of books weighing 22400 lb. for Louisville, Ky. The minimum car load is 24000 lb. The car load rate per 100 lb. was 45¢; the rate per 100 lb. in less than car load lots was $66\frac{1}{2}$ ¢. Which is better for the firm, to pay for a full car load, or to ship at the L. C. L. (less than car load) rate? How much better?

15. A Texas planter in 1919 grew 40 bales of cotton (500 lb. to a bale) for which he received $34\frac{1}{2}$ ¢ a lb. at Houston. The selling and handling charge at Houston was \$1.00 per bale. The freight rate from his shipping point was 20¢ per 100 lb. How much did he get for his cotton crop after paying the freight and charges at Houston?

Advantages of Water Transportation

1. In a recent year 100,000,000 bu. of wheat were carried by steamer from Duluth, Minn., to Buffalo, N. Y., at a cost of 2¢ per bu. The freight by rail between these points was 5¢ per bu. What was the saving in freight on these shipments?

2. It costs \$2.50 per thousand feet to move lumber from northern Michigan and Wisconsin to Buffalo by boat. To ship the same distance by train costs \$5 per thousand feet. If 1,500,000,000 board feet are shipped each year between the points named, what is the saving in freight?

3. In a recent year 100,000,000 tons of freight were carried over the Great Lakes. How many car loads would this make if 20 tons were allowed to each car?



4. The largest inland canal locks in the world are between Lake Superior and St. Mary's River below the rapids. One of these is called the Poe Lock. It is 800 ft. long, 100 ft. wide, and 22 ft. deep. How many boats are handled by this lock during the season of navigation (8 months) if it averages one boat every 14 minutes day and night for the season. (Omit December, January, February, March.)

5. On a map of the United States and Canada, using Buffalo as a center, and a radius representing 500 miles, draw a circle. This circle contains 60% of the population of the United States and Canada. Name at least 12 great commercial centers within this circle. Show how the Great Lakes have been an important factor in building these great cities.

6. Will the shipping on the Great Lakes in the near future become greater, or less, than it is at present? Why?

7. When the L. C. L. rate on a certain commodity from New York to Chicago was \$.63 by rail and water, or \$.788 by all rail, the rate from Chicago by rail to Portland, Oregon, was \$2.00. The same commodity could have been shipped from New York to Portland by an all water route at \$1.25 per 100 lb. What was the comparative cost of shipping by the three routes?

Schedule of Express Rates from Chicago by Scale Numbers
(First Class in Effect Jan., 1920)

Pounds	Scale Numbers							
	13	16	18	23	28	43	124	179
1	\$0.29	\$0.29	\$0.29	\$0.29	\$0.30	\$0.30	\$0.34	\$0.37
2	.30	.30	.30	.31	.31	.33	.42	.47
5	.32	.33	.34	.35	.36	.41	.63	.78
10	.37	.38	.40	.43	.45	.54	.98	1.29
15	.42	.45	.46	.51	.55	.67	1.34	1.79
20	.47	.51	.53	.58	.64	.80	1.69	2.30
25	.52	.56	.59	.66	.73	.93	2.05	2.80
50	.77	.85	.90	1.04	1.18	1.59	3.82	5.33
100	1.26	1.43	1.54	1.81	2.09	2.91	7.37	10.39

Shipping by Express

Commodities shipped by express go more quickly, but at a higher rate, than those shipped by freight. Express charges include free delivery within certain limits. Such shipments are classified as first, second, or third class. The rate on a given commodity varies with the weight of the shipment, and with the distance between the shipping points. The rates charged are determined by the Interstate Commerce Commission.

This commission divided the United States into 826 blocks made by the intersections of the meridians of longitude and

the parallels of latitude. To determine the rate between any block and any other, the agent at the shipping point first refers to a *Directory of Express Stations*, issued jointly by the principal express companies, to find the block number of the destination office, and the scale of rates which applies between his block and the block to which he is shipping. There are 295 different scales in use, each designated by a number.

The agent then refers to his *Schedule of Express Rates by Scale Numbers* and finds what rate applies to the package shipped. This schedule shows rates on packages, weighing from 1 lb. to 100 lb., for each of the 295 scale numbers. The table on page 306 gives a few of these rates for eight of these scale numbers on first class matter. Packages over 100 lb. take the 100 lb. rate.

Some cities to which the scale numbers in the previous table apply on packages expressed out of Chicago.

Fort Wayne, Ind.	} Scale 13	Nashville, Tenn.	} Scale 28
Milford, Ind.		Springfield, Tenn.	
Madison, Wis.	} Scale 16	Boston, Mass.	} Scale 43
Prairie du Sac, Wis.		Haverhill, Mass.	
Louisville, Ky.	} Scale 18	Helena, Mont.	} Scale 124
Carrollton, Ky.		Boulder, Mont.	
Columbus, Ohio	} Scale 23	San Francisco, Cal.	} Scale 179
Chillicothe, Ohio		Fruitvale, Cal.	

1. A Chicago firm shipped the following first class packages by prepaid express in one day:

10 lb. to Ft. Wayne, Ind.	25 lb. to Nashville, Tenn.
2 lb. to San Francisco, Cal.	15 lb. to Columbus, Ohio.
50 lb. to Louisville, Ky.	10 lb. to Boston, Mass.
5 lb. to Helena, Mont.	120 lb. to Madison, Wis.

With the help of the table find the express charge on each package; on all the packages.

2. Find the total express charges out of Chicago on these packages: 10 lb. to Fruitvale, Cal.; 100 lb. to Chillicothe, Ohio; 50 lb. to Milford, Ind.

3. On a certain day in April, 1920, a Chicago firm sent packages of books by express as follows: 50 lb. to Haverhill, Mass.; 20 lb. to Boulder, Mont.; 100 lb. to Columbus, Ohio; 15 lb. to Prairie du Sac, Wis.; 20 lb. to Nashville, Tenn.; 25 lb. to Louisville, Ky.; and 1 lb. to San Francisco. Find the total express charges for the day.

Shipping by Parcel Post

The parcel post law in effect April 15, 1925, provides that packages weighing more than 8 oz. may be sent through the postoffice department at rates which vary for different distances. See the table below. Add 2¢ in each case as "service charge."

The limit of weight for the local and the first three zones is 70 lb.; for all other zones, the limit is 50 lb. A fraction of a pound is counted as one pound.

Table showing rates for all the zones on weights to 6 lb.

		To 50 mi.	50 mi. to 150 mi.	150 mi. to 300 mi.	300 mi. to 600 mi.	600 mi. to 1000 mi.	1000 mi. to 1400 mi.	1400 mi. to 1800 mi.	1800 mi. and over
Wt.	Local	1st	2nd	3rd	4th	5th	6th	7th	8th
1 lb.	\$0.05	\$0.05	\$0.05	\$0.06	\$0.07	\$0.08	\$0.09	\$0.11	\$0.12
2 lb.	.06	.06	.06	.08	.11	.14	.17	.21	.24
3 lb.	.06	.07	.07	.10	.15	.20	.25	.31	.36
4 lb.	.07	.08	.08	.12	.19	.26	.33	.41	.48
5 lb.	.07	.09	.09	.14	.23	.32	.41	.51	.60
6 lb.	.08	.10	.10	.16	.27	.38	.49	.61	.72

The rate of increase shown in the table holds until the largest weight for each zone is reached.

The following formulas are useful in computing the parcel postage. Merely add, in each case, the 2¢ "service charge."

Zone No.	Postage	Zone No.	Postage
Local	$\frac{\text{Wt.}}{2} + 5 = \text{no. } \text{¢}$	1 and 2.	$\text{Wt.} + 4 = \text{no. } \text{¢}$
3.	$\text{Wt.} \times 2 + 4 = \text{no. } \text{¢}$	4.	$\text{Wt.} \times 4 + 3 = \text{no. } \text{¢}$
5.	$\text{Wt.} \times 6 + 2 = \text{no. } \text{¢}$	6.	$\text{Wt.} \times 8 + 1 = \text{no. } \text{¢}$
7.	$\text{Wt.} \times 10 + 1 = \text{no. } \text{¢}$	8.	$\text{Wt.} \times 12 = \text{no. } \text{¢}$

1. The distance from Chicago to Pittsburgh is a little less than 500 mi. What is the postage on an 8 lb. parcel?

2. The distance from St. Louis to Chicago is 292 mi. Find the postage on a package weighing 20 lb.

3. From Chicago to Salt Lake City is 1489 mi. What is the postage on a 4 lb. parcel?

4. Using the formulas on this page, compute the postage on each of these parcel post packages.

Weight	Zone or Distance	Postage	Weight	Zone or Distance	Postage
1. 60 lb.	Local	?	6. 1 lb. 6 oz.	1350 mi.	?
2. 10 oz.	1200 mi.	?	7. 50 lb.	60 mi.	?
3. $1\frac{1}{2}$ lb.	2000 mi.	?	8. $12\frac{1}{2}$ lb.	First	?
4. 40 lb.	Third	?	9. 4 lb.	Sixth	?
5. $10\frac{1}{2}$ lb.	Second	?	10. $5\frac{3}{4}$ lb.	Fourth	?

Solve these with the help of the parcel post and express rates found on pages 306, 307, and 308.

1. Which is cheaper and how much, to send a 10 lb. package from Chicago to Columbus, Ohio, by express or by parcel post? Columbus is in the 4th zone from Chicago.

2. A Chicago firm wishes to send a 15 lb. package of books to Nashville, Tenn., distance about 450 mi. Which is cheaper, and how much, express or parcel post?

3. If the package described in problem 2 had weighed only 3 lb., which would have been the cheaper method of shipment? Which if it had weighed 10 lb.?

Paying Debts at a Distance

In the processes of trade, money or its equivalent must often be sent from one city to another. Instead of the money itself, a check, a draft, or a money order, may be used. Why is this better than to send the money?

1. D. B. Stewart of Chicago, Ill., bought for cash an invoice of goods amounting to \$125 from J. B. Olmstead, Des Moines, Iowa. This is the check which he sent to Olmstead.



(1) What will Olmstead do with the check?

(2) It is said that a canceled check is as good as a receipt. What is meant by this statement?

2. Some firms do not wish to accept personal checks. In such a case a money order (postal or express), or a bank draft (banker's check), may be sent.

Application for Domestic Money Order

Spaces below to be filled in by purchaser, or, if necessary,
by another person for him

Amount

..... Dollars..... Cents

Pay to }
Order of }
(Name of person or firm for whom order is intended)

Whose }
Address }
is } No. Street

Post }
Office }

State

Sent by
(Name of Sender)

Address }
of }
Sender } No. Street

Purchaser must send Order and Coupon to Payee

Fees for Domestic Money Orders

Amt. of Order	Fee
\$0.01 to \$2.50	5 cents
\$2.51 to \$5.00	7 cents
\$5.01 to \$10.00	10 cents
\$10.01 to \$20.00	12 cents
\$20.01 to \$40.00	15 cents
\$40.01 to \$60.00	18 cents
\$60.01 to \$80.00	20 cents
\$80.01 to \$100.00	22 cents

NOTE.—The maximum amount for which a single money order may be issued is \$100. When a larger sum is to be sent additional orders must be obtained. Any number of orders may be drawn on any money order office on any one day.

Roy Brown, Benton, Ohio, bought for cash a baseball glove from Spalding Bros., Cleveland, Ohio, for \$2.00. He sent the money in the form of a postal money order.

1. Before the Benton postmaster issued the money order, Brown wrote an application on a form such as is shown here.

2. How much did he pay the postmaster?

3. What did Brown do with the order received from the postmaster? See page 312 for a form of money order and the receipt.

4. What should Brown do with the receipt? Of what value is it?

5. How did Spalding Bros. get the money for the glove?

6. Write in good form Brown's letter to Spalding Bros.

10277 Chicago, Sta. 126, Ill. 91830
OFFICE NUMBER SERIAL NUMBER

United States Postal Money Order

June - 4 - 1921

THE POSTMASTER AT *Cleveland Ohio*

WILL PAY AMOUNT STATED ABOVE TO ORDER OF PAYEE NAMED IN ATTACHED COUPON OF SAME NUMBER. IF ISSUED WITHIN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED, THE POSTMASTER AT ANY MONEY ORDER OFFICE IN THE CONTINENTAL UNITED STATES, ALASKA EXCEPTED, WILL PAY IF PRESENTED WITHIN THIRTY DAYS FROM DATE OF ISSUE.

PAYEE'S NAME *Spalding Bros*

AMOUNT *2* DOLLARS *4* CENTS

POSTMASTER *R. B. Barile*

RECEIVED PAYMENT

STAMP HERE

PAYING OFFICE

PAYING POSTMASTER DETACH COUPON ON THIS LINE

The purchaser must send both the order and the coupon to person named in the coupon as payee.

Express Money Orders

The fee for these order's is the same as that for postal money orders. No single order will be issued for more than \$50. The number of indorsements is not limited to one.

Chicago, Sta. 126, Ill. 91830
OFFICE NUMBER SERIAL NUMBER

Coupon for Paying Office
NOT TO BE DETACHED BY HOLDER

PAYEE: WRITE WORDS FOR DOLLARS *Two* DOLLARS *4* CENTS

REMITTER: *Ray Brown*

THIS MONEY ORDER IS NOT GOOD FOR MORE THAN LARGEST AMOUNT INDICATED ON LEFT-HAND MARGIN OF THE ORDER AND ANY ALTERATION OR ERASURE RENDERS IT VOID

ISSUING OFFICE

STAMP HERE

91830
RECEIPT

DOLLARS *2* CENTS *4*

AMOUNT FOR WHICH ISSUED

TO BE DETACHED BY THE PURCHASER, WHO SHOULD PRESENT IT AT THE OFFICE OF ISSUE IF HE WISHES TO MAKE INQUIRY REGARDING THE ORDER

ISSUING OFFICE

STAMP HERE

EXPRESS MONEY ORDER

WHEN COUNTERSIGNED BY AGENT AT POINT OF ISSUE

D-747698

American Express Company

AGREES TO TRANSMIT AND

Pay on presentation to *Spalding Bros* or order *2* DOLLARS *4* CENTS

The Sum of *Two and 4* CENTS

Countersigned *A. F. Barile* Agent

Issued at *Chicago, (Branch) State of Ills.*

Date *6-4* 1921

NAME OF REMITTER *Ray Benton*

TREASURER: *R. B. Barile*

ANY ALTERATION OR ERASURE OR DETACHMENT OF THIS ORDER RENDERS IT VOID

Business men find it more economical to remit by **bank draft** than by money order, if they cannot send their personal check without adding an extra 15¢ or 25¢ for exchange.

A draft is an order by one bank on another bank to pay a certain sum of money to a party named in the draft.

The Farmers' and Merchants' Bank

Millersburg, Ohio, July 1, 1920.

Pay to the Order of	<u>John Gibbs</u>	<u>\$125.00</u>
	<i>One Hundred Twenty-five</i>	Dollars
To	<i>Central National Bank,</i>	<i>Louis Miley</i>
	<i>Cleveland, Ohio</i>	Cashier

BANK DRAFT

John Gibbs is a farmer near Millersburg, Ohio, who owes the Cleveland Milling Co. a bill for mill feed, amounting to \$125. Since he has an account with the Millersburg bank, he is not charged a fee for drafts which he may wish to buy at this bank.

1. Of what bank is Louis Miley cashier?
2. Who is ordered to pay \$125? Who pays \$125 to the Millersburg bank?
3. Why can the Millersburg bank order the Cleveland bank to pay \$125?
4. To whom is this draft payable?
5. What must Gibbs do before sending the draft to the Cleveland Milling Co.?

Commercial Drafts

A **commercial draft** is a written request directing a person or firm to pay to a bank, or person, a certain sum of money at a stated time.

Adam Miller of Mt. Liberty, O., owes Brown & Co. of Columbus, O., \$140 on account. If Miller is not prompt in paying the account when it is due, Brown & Co. may draw on him, using the following commercial draft.

<u>\$140.00</u>	Columbus, Ohio, <u>Dec. 15, 1920</u>
<u>At Sight</u> Pay to the order of the <u>New First National Bank</u>	
<u>One Hundred Forty</u>	<u>Dollars</u>
Value Received and Charge to the Account of	
To <u>Adam Miller,</u>	
<u>Mt. Liberty, O.</u>	<u>Brown & Co.</u>

Brown & Co. deposit this draft with their bank (New First National) for collection. After endorsing it this bank sends the draft to a bank at Mt. Liberty, which presents it to Miller for *acceptance*. If he accepts the draft by writing the word "Accepted," the date, and his signature across the face of it, the draft is good, and being a sight draft it becomes due at once. Miller may give his check for \$140 in favor of the Mt. Liberty bank. This bank in turn remits to the Columbus bank, which places the amount, less a small fee for collection (usually $\frac{1}{4}\%$), to the credit of Brown & Co.

Sight drafts are due when they are accepted.

Time drafts are due a stated number of days after date or after acceptance. The draft must state definitely when

payment is to be made. The draft on this page is due 60 days after it is accepted.

\$3170.45	NEW YORK, N. Y., <u>Mar. 16, 1919.</u>
~~~~~ <i>At sixty days sight</i> ~~~~~	PAY TO THE ORDER OF
~~~~~ <i>Chas. G. Price</i> ~~~~~	
<i>Thirty-one Hundred Seventy and ⁴⁵/₁₀₀</i> ~~~~~	DOLLARS
VALUE RECEIVED AND CHARGE TO ACCOUNT OF	
To <i>Mabley-Carew Co.;</i>	
<i>Cincinnati, Ohio.</i>	<i>Chas. G. Price</i>

Problems

1. The draft on page 314 was accepted on Dec. 17. Copy it and write the proper acceptance across the face.
2. Write the draft on this page. Accept it on Mar. 24.
3. When is the draft in example 2 due? Who pays it? To whom?
4. Write a commercial time draft due in ten days after sight, in which Brown & Co. draw on Frank Smith of New London, O.
5. Sidney Brown of Madison, Ind., owes Lyons and Carnahan of Chicago, Ill., \$60 due Oct. 1, 1920. Write the proper commercial sight draft. Who signs this draft? Trace the draft until Lyons and Carnahan are credited with \$60, less 10¢ for collection.

CHAPTER X

SAVINGS ACCOUNTS

Many banks have a savings department in which a low rate of interest (3% or 4%) is paid on savings deposits. Such an account may be opened with one dollar. Interest is generally computed and added to the account on Jan. 1 and July 1 of each year. The rules governing interest on savings deposits, followed by many banks, are these:

1. On deposits made during the first ten days of January and July, and during the first five days of any other month, interest will be allowed from the first day of the month in which such deposit is made. On deposits made after the above named days of each month, interest will be allowed from the first day of the month following the deposit.

2. Interest will not be allowed on the average balances, nor for parts of months, nor for parts of dollars, nor on sums withdrawn between January 1 and June 30, or between July 1 and December 31. In computing interest, withdrawals between these periods will be deducted from the first deposits.

EXAMPLE.—A man deposits \$100 January 2, also \$100 on June 1. On June 15 he withdraws \$100. Under the rule stated above this man receives interest on July 1 on \$100 for one month.

Problems

1. This problem shows how the above rules are used.

On Jan. 6, 1920, James Baker deposited \$100 in a savings bank which paid 4% per annum, interest payable semi-annually. On Mar. 6 he deposited \$50. On Aug. 9 he deposited \$25. On Nov. 24 he withdrew \$35. What balance was due Jan. 1, 1921?

James Baker

Date	Deposits	Interest	Withdrawals	Balance
1920				
Jan. 6	100			100
Mar. 6	50			150
July 1		2.50		152.50
Aug. 9	25			177.50
Nov. 24			35	142.50
1921				
Jan. 1		2.67		145.17

EXPLANATION.—The interest is computed and entered on July 1 and January 1, and then it becomes a part of the balance. On July 1, 1920, Baker was entitled to interest on \$100 for 6 mo. (\$2.00) and on \$50 for 3 mo. (50¢). See Rule 1. This interest was added to the balance on July 1.

On January 1, 1921, he was entitled to interest on \$117 (\$152.50—\$35 Rule 2) for 6 mo. and on \$25 for 4 mo. The interest due on Jan. 1, 1921, was \$2.67, which made the balance on this date \$145.17.

2. Find Baker's balance on July 1, 1921, if he deposited \$40.75 on Mar. 2, 1921, and withdrew* \$50 on June 10.

3. Was it good business for Baker to draw on his savings account on June 10 if he could have borrowed the money for a month at 8% per annum?

* NOTE.—If you would have your savings account earn the maximum amount of interest, observe these three rules:

(1) Make your deposits in whole dollars before the 5th of each month. Why?

(2) In the first week of January and July, add enough to the interest to make the next dollar. Why?

(3) Make no withdrawals except at interest periods. Why?

4. A young woman had a balance of \$52 in a savings bank on Jan. 1. Her deposits were as follows: Feb. 12,

\$10; Mar. 3, \$15; May 1, \$20; June 1, \$5. What was her balance on July 1 if interest at 4% per annum, payable semi-annually, is allowed?

5. A young man owns some Liberty Bonds. He put all the interest from them into his savings account at 4% per annum, payable Jan. 1 and July 1. These were the amounts of interest he collected in a certain year:

Mar. 15,	\$ 8.50	Sept. 15,	\$ 8.50
Apr. 15,	4.25	Oct. 15,	4.25
May 15,	17.00	Nov. 15,	17.00
June 15,	4.00	Dec. 15,	4.00

How much interest did his Liberty Bond interest earn for him that year?

Compound Interest

In your study of savings accounts you found that banks added the interest at certain periods, usually Jan. 1 and July 1, to the balance of the account at those periods. Interest for the next period was computed on this sum. In other words, interest was allowed on the principal and on the interest due.

This kind of interest is called compound interest. Adding the interest due at a certain period to the principal and computing interest on this sum is called compounding interest. Interest may be compounded annually (once a year), semi-annually (every six months), or quarterly (every three months). Banks which allow interest on the **checking account** add to the account at the end of the month the interest earned during the month. This is compounding interest monthly.

Compound interest is legal in several states* if specified in the contract. In the rest of the states compound interest cannot be collected, even if called for in the contract. There is no penalty attached to paying compound interest by those who wish to do so for business reasons. If savings banks did not allow compound interest, depositors might draw their interest when due and with it start a new account, which would amount to the same thing as getting compound interest.

Compound Interest Problems

1. This table shows how money increases if compound interest is allowed.

At 3%, interest added annually.

\$100 during the first year becomes \$103.

(How did we get \$103?)

The \$103 during the second year becomes \$106.09.

(How did we get \$106.09?)

The \$106.09 during the third year becomes \$109.27.

(How did we get \$109.27?)

The \$109.27 during the fourth year becomes \$112.55.

The \$112.55 during the fifth year becomes.....

The during the sixth year becomes

(1) Find the values at the end of the fifth and sixth years in the table.

(2) The compound interest on \$100 for 6 years at 3% is \$..... less \$100.

(3) How much greater is the compound interest which you found in the previous problem than the simple interest for the same time and rate?

* Cal., Iowa, Mo., Mont., Neb., Tenn.

2. Find the compound interest for 2 years on \$200 at 4% per annum payable semi-annually.

HINT.—At the end of the first 6 months, the \$200 would be \$204. At the end of the first year the \$204 would be \$208.08. At the end of $1\frac{1}{2}$ years the \$208.08 would be, etc. The compound interest for 2 yr. = the amount at the end of the second year less \$200.

Persons who must compute compound interest often do so by means of a table, a part of which is shown here.

Compound Interest Table

Interest Period	RATE OF INTEREST FOR THE PERIOD						
	1%	$1\frac{1}{2}\%$	2%	$2\frac{1}{2}\%$	3%	4%	6%
	AMOUNT OF \$1.00						
1	1.0100	1.0150	1.0200	1.0250	1.0300	1.0400	1.0600
2	1.0201	1.0302	1.0404	1.0506	1.0609	1.0816	1.1236
3	1.0303	1.0457	1.0612	1.0769	1.0927	1.1249	1.1910
4	1.0406	1.0613	1.0824	1.1038	1.1255	1.1699	1.2625
5	1.0510	1.0772	1.1041	1.1314	1.1593	1.2167	1.3382
6	1.0615	1.0934	1.1262	1.1597	1.1941	1.2653	1.4185
7	1.0721	1.1098	1.1487	1.1887	1.2299	1.3159	1.5036
8	1.0829	1.1265	1.1717	1.2184	1.2668	1.3686	1.5938
9	1.0937	1.1434	1.1950	1.2489	1.3048	1.4233	1.6895
10	1.1046	1.1605	1.2190	1.2801	1.3439	1.4802	1.7908

How to Construct the Table

1. The amount for each succeeding period is found by computing the interest at the given rate on the last known amount and adding the interest found to this amount.

The amount for the 9th period at 1% is \$1.0937. The interest for the next period (10th) is 1% of this amount, which is \$.0109. The amount for the 10th period = \$1.0937 + \$.0109 = \$1.1046.

2. Make the table for the 11th period; for the 12th.

How to Use the Table

1. If the interest is compounded annually at 6% the amount due at the end of 5 years is found by looking in the 6% column opposite the figure 5 in the period column. This amount (\$1.3382) - \$1.00 is \$.3382, the compound interest on \$1 at 6% for 5 years. How would you find the compound interest for \$500 at 6% for 5 years?

2. If the interest is compounded semi-annually, take half of the given rate and twice the number of years and proceed as if the interest were compounded annually.

Find the compound interest on \$1 at 4% for 5 years compounded semi-annually. Look in the 2% column opposite 10 in the period column. The amount is \$1.2190. $\$1.2190 - \$1.00 = \$.2190$, compound interest of \$1 for 5 yr. at 4% compounded semi-annually.

3. If the number of periods is greater than 10, the largest shown in the table, find the amount of \$1.00 for 10 periods at the given rate; then find the amount for the remaining time at the same rate and multiply these two amounts together. The product is the amount of \$1.00 for the entire time.

The amount of \$1 at 3% for 15 years compounded annually is \$1.5580. Prove this result by multiplying 1.3439 (10 period amount) by 1.1593 (5 period amount).

4. If the time contains a fraction of an interest period, add to the amount for the last full period, such a part of the interest for the next full period as is indicated by the fraction of the time period.

The compound amount of \$1 at 6%, compounded annually, for $2\frac{1}{2}$ years is $\$1.1236 + \frac{1}{2}$ of $(\$1.1910 - \$1.1236)$, which is \$1.1573.

Practice in Using the Compound Interest Table

1. A boy saved \$100 during 1909. On Jan. 1, 1910, he invested this money at 3%, compounded annually. What was the amount of his investment on Jan. 1, 1920?

2. Some 8th grade pupils bought in Jan., 1920, War Savings Stamps at \$4.12 each. On Jan. 1, 1925, the United States Government will pay \$5.00 for each of these stamps. Some time later the teacher said to the pupils, "You are receiving on your stamp investment 4% interest, compounded annually for 5 years." Use the interest table to test the accuracy of this statement.

3. In July, 1920, some of the pupils mentioned in the previous problem bought War Savings Stamps at \$4.19 each. These stamps are worth \$5.00 each on Jan. 1, 1925. Use the table to find out if this investment pays 4% interest, compounded annually.

4. Can you solve the previous problem without using the table? Try it. Compare the two solutions.

5. A teacher in Nov., 1920, bought 5 War Savings Stamps at \$4.22 each. These stamps are worth \$5.00 each on Jan. 1, 1925. Find out if the buyer made a 4% interest investment, compounded annually.

6. Which will pay more, and how much, \$100 at 3% for 20 years at simple interest or \$100 at $2\frac{1}{2}\%$ for 20 years, compounded annually?

7. A thrifty young man invested on Jan. 1, 1916, \$300 in a business paying 4% interest, compounded annually. At the time of each interest payment except the last he added \$300 to his investment. What was due 1/1, 1921?

CHAPTER XI

ECONOMY AND THRIFT

This chapter contains problems intended to show how a knowledge of arithmetic enables one to practice economy and thrift, by helping to answer the question, "Will it pay?" or "Did it pay?" or "Which course of action is most economical?"

Thrift in the School

1. The children of the Mullanphy School collected in two months 11325 lb. of old newspapers and 2550 lb. of magazines. They received \$1.25 per 100 lb. for the newspapers and \$2.75 per 100 lb. for the magazines. What was the total received for old paper?

2. The children of the school named in the previous problem collected from Sept. 1, 1919, to Jan. 1, 1920, 17975 lb. of old magazines. What was the average amount received per month at the price named in the previous problem?

3. The children of the public schools of St. Louis during March and April, 1920, collected, weighed, and computed the value of 267450 lb. of old newspapers and 28514 lb. of old magazines. Newspapers were priced at \$1.25 per 100 lb.; magazines at \$2.75 per 100 lb. Find the total value.

4. If the children had not collected this old paper, what would probably have happened to most of it? Name several ways in which collecting and selling old paper promotes thrift?

Knowing Food Values



Each of the above amounts equals 1 qt. of milk in energy value.

Table of Jan., 1921, Prices

Milk . . .	16¢ a qt.	Tomatoes .	30¢ a lb.	Fish (halibut)	35¢ a lb.
Eggs . . .	75¢ a doz.	Bananas . .	8¢ a lb.	Sirloin steak	40¢ a lb.
Cabbage . .	4¢ a lb.	Oranges . .	60¢ a doz.	Eng. walnuts	28¢ a lb.

1. If you use the cost of a quart of milk as the basis, which is the cheaper food at the prices stated in the table; oranges or bananas? eggs or sirloin steak? English walnuts or fish? eggs or fish?

2. Which is the most expensive food? Which is the cheapest at the prices quoted?

3. Calling the energy* value of 1 qt. of milk one, or 100%, find the % of energy value in 1 lb. of each of the other foods in the figure.

* Energy is ability to do work.

4. If eggs were 36¢ a dozen, what could you afford to pay for a quart of milk to get an equivalent energy value? What with oranges at 40¢ a dozen? What with bananas at 5¢ a lb.? What with cabbage at 2¢ a lb.? What with sirloin steak at 35¢ a lb.?

5. Make and solve other problems based on the foods shown in the chart on page 324, using current prices.

6. A pound of crackers equals $1\frac{1}{2}$ lb. of bread in energy value. Why is this true? Compare their costs.

7. A pound of rice in energy value equals a pound of macaroni. Compare their costs.

8. When a certain variety of apples was retailing at 10¢ a lb., the same variety at a special sale cost only \$2.85 per box of 42 lb. net. What was the % of saving when bought by the box?

9. In Oct., 1919, potatoes sold in a large city at \$2.00 per bu. In March, 1920, they sold at 10¢ a lb. What % was saved on every pound, used in March, but bought in October?

10. People who have storage room for potatoes and apples usually buy their winter supply in the autumn. Do you see why?

11. Canned corn which retails at 15¢ per can is usually sold at \$1.50 per dozen. If a family needs 3 dozen cans, what is the % of saving by buying at wholesale?

12. In May, 1920, the price of old potatoes was 10¢ a lb.; of new ones it was 20¢ a lb. If the larger waste in peeling old potatoes is offset by their higher food value, what is the % of saving in buying old potatoes?

Home and School Problems

1. When sweet potatoes were selling at 11¢ per lb., Irish potatoes sold at 10¢ a lb. In energy value 1 lb. of sweet potatoes equals $1\frac{1}{2}$ lb. of Irish potatoes. What % is saved for each pound of sweet potatoes bought?

2. In May, 1920, a 48-lb. sack of flour sold at \$4.10, and a 12-oz. loaf of bread sold at 10¢. What was the % of saving by baking at home if it requires 9 oz. of flour, $\frac{1}{2}$ ¢ worth of other ingredients, and $\frac{3}{4}$ ¢ worth of fuel for each loaf? Carry results to tenths of a mill.

3. At the prices stated in the previous problem how much money does the housewife save by baking at home for each 48-lb. sack of flour used? Allow 1% of the flour as waste in making the dough.

4. In a large city during 4 months of 1919 the school children collected 682002 lb. of newspapers and 78345 lb. of magazines.

(1) How many tons were there in the total?

(2) If during this period the average price of old newspapers was \$1.15 per 100 lb., what was the value of the average monthly collection?

(3) The average price for old magazines was \$2.45 per 100 lb. Find the value of the average monthly collection.

(4) Find the total value of old paper collections for the four months.

(5) At the rate for the first four months what would be the value for the next five?

(6) At this rate what would be the value for the nine calendar school months?

A Missouri teacher bought for \$200 an acre of land containing the following apple trees: Rome Beauty 14; Arkansas Black 16; Ben Davis 20. In 1919 the total production and value were as follows: Rome Beauty 10 bbl. No. 1 @ \$3.75, 6 bbl. No. 2 @ \$3.00; Arkansas Blacks 10 bbl. No. 1 @ \$3.75, 8 bbl. No. 2 @ \$3.00; Ben Davis 12 bbl. No. 1 @ \$3.50, 6 bbl. No. 2 @ \$2.75; 8 bbl. of windfalls @ \$2.50; windfalls sold in baskets and sacks for \$11.50; 10 gallons of cider at 35¢; culls fed to 4 hogs for 2 months, estimated at \$4 per hog.

The cost of growing and marketing the crop was as follows: pruning, 3 days @ \$2; spray materials, \$7; spraying, 4 times, requiring each time 2 men working 4 hours, at 25¢ per hr.; picking, sorting, and packing, 40¢ a bbl. (omit the windfalls not in barrels); empty barrels, 49¢ each; drayage to freight station $\frac{1}{4}$ mile from the orchard, \$4.50.

Compute the interest on the cost of the land at 6%. The tax was \$1.60 per \$100 on an assessed value of \$150.

1. Find the total receipts from the orchard.
2. Find the average receipts per tree.
3. Find the total cost of the crop including interest on the land and the tax.
4. Find the average cost per tree.
5. Find the net earnings of the orchard.
6. Find the net earnings per tree.
7. What % of the entire cost is the cost of picking and marketing (3 items)?
8. What % net did the orchard earn?
9. Is it safe to expect such a crop as this each year?

Thrift in Growing Corn

I. This is the record of the cost and production of one acre of corn as kept by a Missouri corn-club boy. Find the value of each question mark.

Date		Record of Work and Cost*					Value
Mar.	3	2 hr. making seed tester @ 15¢.	.	.	.	.	?
"	18	1 hr. selecting and testing seed	.	.	.	.	?
"	18	Cost of seed	.	.	.	.	\$0.50
"	30	" " fertilizer	.	.	.	.	2.50
Apr.	3	4 hr. labor, boy and team, plowing @ 65¢	.	.	.	.	?
"	15	1 hr. " " " " harrowing	.	.	.	.	\$0.65
May	10	2 hr. " " " " planting @ 65¢	.	.	.	.	?
"	25	1 hr. " " " " harrowing	.	.	.	.	?
"	28	1 hr. thinning	.	.	.	.	\$0.15
"	29	1½ hr. labor, boy and team, cultivating @ 65¢	.	.	.	.	?
June	4	1½ hr. " " " " " @ 65¢	.	.	.	.	?
"	15	1½ hr. " " " " " @ 65¢	.	.	.	.	?
"	25	1½ hr. " " " " " @ 65¢	.	.	.	.	?
July	3	1½ hr. " " " " " @ 65¢	.	.	.	.	?
Oct.	30	12 hr. husking @ 15¢	.	.	.	.	?
"	31	3 hr. hauling @ 65¢	.	.	.	.	?
Nov.	15	Rent on land	.	.	.	.	\$5.00
Total cost of growing one acre							?
Nov.	2	Sold 68 bushels @ \$1.20	.	.	.	.	?
Net profit on one acre							?

* NOTE.—The above record is made in full to give city children and others a reading knowledge of scientific corn growing as practiced by thousands of boys in our country.

II. This is a summary of an account kept by an Ohio farmer with a 10-acre field of corn in 1918.

March	Cost of seed corn, \$5. Testing seed, 8 hr. @ 25c. Fertilizer, \$25.
April	Plowing, 1 man and team, 66 hr. @ 75¢.
May	Harrowing, 20 hr. @ 75¢. Planting 10 hr. @ 75¢.
May, June, July	Cultivating 4 times, each cultivation requiring 15 hr. @ 75¢.
October	Gathering 610 bu. shelled corn, 75 hr. man and team @ 75¢; helper, 75 hr. @ 25¢.
December	Interest at 6% for 6 months on the land worth \$150 per acre. Sold the crop in the crib at \$1.40 per bu.

(1) Find the gross profit.

(2) What item not shown above must be subtracted to find the net profit?

(3) Suppose the field had yielded only 30 bushels per acre, due to an August drouth, and the market price had been \$1.00 per bushel, what would have been the gross profit in the above problem?

(4) Find the net profit at the current price of labor and of corn.

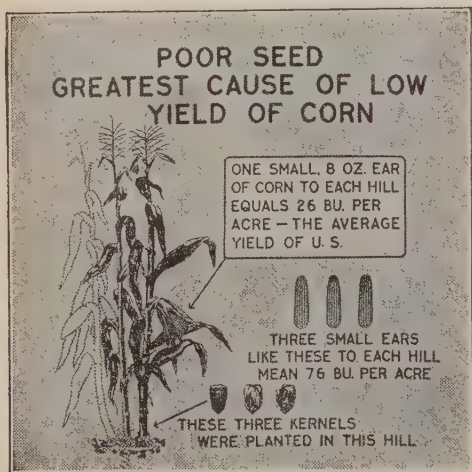
(5) Is corn a profitable crop in your community?

(6) Is there a close relation between the price of hogs and of corn? Between the price of hogs and of wheat? See the market columns of the newspaper.

III. In a certain experiment it was found that 112 grains out of 172 grains of corn six years old sprouted. The same corn when one year old made 98% in a sprouting test. What was the loss of vitality in per cent in 5 years?

Sometimes corn one year old sprouts (germinates) only $\frac{1}{3}$ of the grains planted. What lesson may a careful farmer learn from the experiment described on page 329?

IV. Study the chart, and from it solve the problems below.



(1) How many hills are planted on an acre if an 8-ounce hill yields 26 bu. to the acre? (A bushel of ear corn weighs 70 pounds.)

(2) What is the average weight per ear of the small ears in the picture which yield 76 bu. to the acre?

(3) Fred Dobbins of Cedarville, O., in a recent year averaged 104 bushels of corn to the acre. On a basis of 3 ears to the hill, what was the average weight per ear?

(4) The difference between a good yield of corn and a poor one, with the same soil and climatic conditions, is largely the difference between good seed and poor seed. When should seed corn be selected? Why should it be tested before planting?

V. If corn sold at \$1.50 per bushel on Oct. 1, 1918, and the same quality sold at \$1.80 on July 1, 1919, how much

a bushel was made by holding until July? Allow 1% a month for shrinkage and other loss in the crib, and 6% interest on the money from October to July.

I. James McMillen, an Ohio pig-club boy, purchased on May 23, 1918, a 37-pound Poland-China pig, 2 months old. Later weights were as follows: July 8, 112 lb.; Sept. 12, 229 lb.; Nov. 2, 378 lb.; Dec. 21, 515 lb.



(1) Find the average daily gain for each period.

(2) During which period was the per cent of gain greatest?

(3) Construct a graph showing the growth from May 23 to Dec. 21.

II. Another Ohio pig-club boy raised a pig whose weight record was as follows: 3 months old, 40 lb.; 6 months old, 110 lb.; 9 months, 220 lb.; 12 months old, 300 lb.

(1) During which 3 months' period did the pig make the highest % of gain?

(2) During which period the lowest % of gain?

(3) During which period did the pig make the largest average daily gain?

(4) If it required 12 bu. of corn and \$2.50 worth of other feed to raise the above pig until it was 6 months old, at the current price of hogs and corn did the pig show a profit, or a loss, at 6 months of age?

(5) It required 10 bu. of corn worth \$1.50 a bu. and \$2.00 worth of other feed to feed the pig between the age of 6 months and 9 months. Did this period show a profit or a loss if hogs were worth 17¢ a lb.?

(6) Compare the rate of growth of this pig with the Poland-China in I. Both pigs received the same care and the older one consumed more feed. How do you account for the difference?

III. In dry lot feeding, 7 lb. of shelled corn are allowed daily to a 200-lb. pig. On clover pasture only $\frac{1}{2}$ this amount of corn is necessary to get the same gain.

(1) What is the daily value of the clover pasture for each 200-lb. pig?

(2) If the daily increase is $1\frac{1}{2}$ lb. in weight, how much per bushel is the owner getting for the corn fed in the dry lot, not counting his labor?

IV. A balanced ration for growing pigs is 6 lb. of shelled corn, 3 lb. of shorts, and 1 lb. of oilmeal. Skimmed milk, or buttermilk, may replace partly or wholly the shorts and oilmeal on the basis of 3 lb. of milk to 1 lb. of corn.

(1) If a boy can procure daily 24 lb. of skimmed milk, how much corn will balance this amount?

(2) How much shorts and oilmeal must the boy add to the milk to balance a daily ration of 12 lb. of corn to get the best results?

V. A pig with ordinary care and proper feed (12 bu. of corn and \$2.50 of other feed), weighing 115 lb. at 6 months of age, is a "scrub hog." A thoroughbred at the same age with the same care and feed weighs on an average 200 lb.

(1) At current prices of hogs and corn, what is the difference in profit between the two types?

(2) What does the farmer mean when he says, "My pigs are eating their heads off"? How may he remedy this condition?

VI. It is estimated that 100 lb. of shelled corn fed to a hog will increase its weight 20 lb.

(1) If hogs are worth \$16 a hundred-weight and corn \$1.30 per bu., would a farmer gain, or lose, by feeding his crop of corn to his hogs? 56 lb. of shelled corn = 1 bu.

(2) If his crop is 800 bushels, find the gain or loss.

(3) The above estimate is based on dry lot feeding. If the corn is fed in clover pasture, only half the amount of corn named above is needed to make 20 lb. of increase. Use the information in this section to help you in the next one.

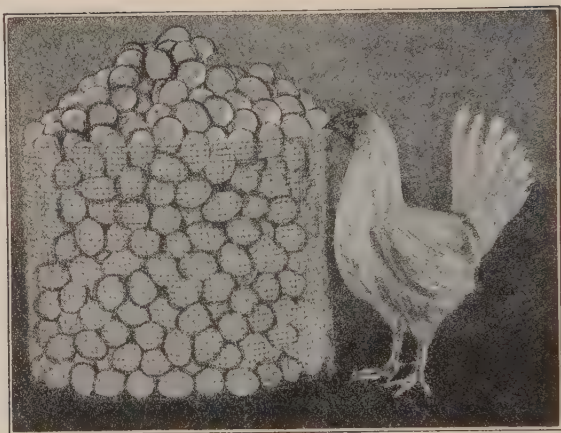
VII. In July, 1919, good hogs sold at \$22 per hundred-weight and corn was worth \$1.80 per bushel.

(1) What profit on his corn did the farmer make each day, who fed 100 bushels of shelled corn in 20 days to July hogs in a dry lot?

(2) How many hogs was he feeding if he allowed each hog 7 lb. of shelled corn daily?

(3) How many might he have fed in clover pasture with this amount of corn? Would his profit have been greater by feeding in clover? Why?

VIII. A hog weighing 650 lb. on foot was sold in 1919 at 22¢ a lb. What do you think about the size of the hog and the price per lb.?



Keeping Poultry for Profit

I. Missouri Queen was a Single-Comb White Leghorn hen at the Missouri Poultry Experiment Station. This remarkable hen completed her 1000-egg record April 10, 1919. This is her record: 1914, 222 eggs; 1915, 187 eggs; 1916, 217 eggs; 1917, 149 eggs; 1918, 177 eggs. She weighed $4\frac{1}{2}$ lb., and it cost on an average the value of 110 eggs to feed her each year. The weight of the eggs was 121.8 lb.

(1) Find the number of eggs laid in 1919 to complete the 1000-egg record.

(2) If the record up to April 10 was continued through the year, find the 1919 egg production.

(3) Construct a graph showing the egg record for 6 years.

(4) Compute the % of profit earned for her owner each year.

II. This table shows a one-year egg record kept by Miss Bertha Stanford, age 15, an Atchison Co. (Mo.) poultry-club girl.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
1.	0	6	1	17	14	13	13	19	16	16	3	0	?
2.	0	3	3	16	15	13	13	18	15	16	2	0	?
3.	0	2	5	15	14	14	14	18	15	16	2	0	?
4.	0	1	6	15	14	14	14	18	14	15	2	0	?
5.	0	0	8	15	14	15	13	17	14	15	1	0	?
6.	0	0	9	18	13	15	14	17	14	15	1	0	?
7.	0	0	10	18	14	15	14	17	14	15	1	0	?
8.	0	0	12	18	13	16	15	17	14	14	0	0	?
9.	0	0	13	13	14	16	16	18	16	13	0	0	?
10.	0	0	15	13	13	16	18	17	15	11	0	0	?
11.	0	0	15	13	13	16	20	17	15	11	0	0	?
12.	0	0	15	13	13	15	18	18	15	11	0	0	?
13.	0	0	17	12	14	15	18	18	16	11	0	0	?
14.	0	0	17	15	15	14	18	18	16	11	0	0	?
15.	0	0	17	15	16	15	18	18	16	10	0	0	?
16.	0	0	19	16	16	14	19	17	16	9	0	0	?
17.	0	0	20	16	16	13	19	17	16	9	0	0	?
18.	0	0	20	15	16	13	19	18	16	9	0	0	?
19.	1	0	20	14	15	13	18	17	16	7	0	0	?
20.	2	0	19	15	15	14	18	16	17	7	0	0	?
21.	2	0	19	14	16	14	18	16	18	7	0	0	?
22.	3	0	20	14	15	14	19	15	18	6	0	0	?
23.	4	0	17	14	14	15	19	15	18	6	0	0	?
24.	3	0	16	13	14	15	20	14	19	6	0	1	?
25.	4	0	17	13	14	15	20	15	18	5	0	2	?
26.	5	0	16	14	14	16	19	15	19	4	0	2	?
27.	5	0	18	15	13	16	20	16	18	5	0	2	?
28.	5	0	17	15	13	15	19	16	17	4	0	3	?
29.	5		18	15	13	14	18	16	17	4	0	3	?
30.	6		18	14	14	14	19	16	17	3	0	4	?
31.	6		18		0		19	16		3		5	?
Total	?	?	?	?	?	?	?	?	?	?	?	?	??

(1) What was the monthly production? The total daily production?

(2) Find in two ways the total number of eggs laid during the year.

(3) Construct a graph showing the relative monthly yield.

(4) In 1918 the average price of eggs per dozen for each month beginning with Jan. was as follows: 60¢, 55¢, 40¢, 34¢, 30¢, 29¢, 27¢, 29¢, 31¢, 35¢, 50¢, 62¢. At these prices find the value of each month's egg production.

(5) Construct a graph, showing the relative monthly value of eggs laid. Construct a graph, showing the monthly fluctuation in the price per dozen.

(6) Miss Stanford reported the cost of the feed for the year to have been \$20. What was the excess of egg value over feed cost?

(7) Study the egg record once more, take into consideration the feed cost, then suggest a possible method of increasing the egg production during the winter months. Why is such an increase desirable?

III. In March, 1918, James Vaughn, age 12, a Monroe Co. (Mo.) poultry-club boy, went into the poultry business. He built two houses, costing \$25, and one coop, costing \$5.00. He paid \$1.50 for 5 doz. hatching eggs, \$5.00 for 8 laying hens, and \$10.00 for feed. He sold 8 head of market poultry @ 80¢, 10 fowls for breeding purposes @ \$1.00, and 30 doz. market eggs at an average of 45¢ per doz. On Nov. 1, he had on hand his poultry houses, 10 young roosters, 8 hens, and 17 pullets. He valued the roosters at 85¢ each, the hens at \$1.00, the pullets at \$1.25. He allowed 20% for depreciation on his poultry houses.

(1) Make an account showing this boy's financial condition on Nov. 1 with regard to his poultry project.

(2) What in your judgment should the boy have done with the young roosters before Dec. 1?

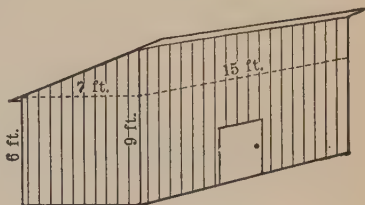
IV. This is a cut of Harry's poultry house. The roof projects 6 inches over each side.

(1) How long must Harry cut the boards for the roof?

(2) How much lumber one inch thick will be needed for the roof and sides?

(3) What other lumber did he need?

(4) How much will it cost to paint the entire outside surface at 15¢ a square yard?



Mixed Problems

1. How many thrift 25-cent stamps can Henry buy with a year's interest from his \$100 bond if it earns $4\frac{3}{4}\%$?

2. A family spends \$600 for rent, \$2400 on other expenses, and saves \$600. If they increase their total expenses to \$3500 and their savings in the same ratio, how much will they save?

3. Three girls start a lemonade stand. The first invests 50¢, the second 75¢, and the third \$1.25. At the end of the day they have a profit of \$3. What is each girl's share?

4. A certain brand of soup sells at 13¢ for 11 oz. net. Another brand equally good sells at 15¢ for 17 oz. net. What is the rate of saving by buying the latter?

5. After cutting from a $12\frac{1}{2}$ lb. cheese three pieces which were said to weigh $2\frac{3}{4}$ lb., $1\frac{1}{2}$ lb., and $\frac{3}{4}$ lb. respectively, it was found that exactly $7\frac{1}{4}$ lb. remained. On the average how nearly accurate was the clerk who weighed the pieces?

6. Some bolts of cloth contain exactly 50 yards. After cutting pieces which were said to measure as follows: 3 yd., $2\frac{1}{2}$ yd., $6\frac{3}{4}$ yd., $\frac{1}{2}$ yd., 10 yd., and $4\frac{1}{4}$ yd., exactly $22\frac{3}{4}$ yd. were left. If the cloth was worth \$2.40 a yard, how much, due to careless measuring, on the average did it cost the store each time a piece was sold? Do you see why some clerks in a large store are promoted more rapidly than others?

7. A young working man earning 50 cents an hour took a 6-months' business course at night school. The tuition was \$60, including all the necessary books and stationery. Car fare cost him 10¢ a day. In order to do this, he lost one hour daily from his work. How much did the course cost him?

8. On Feb. 7 the reading on my gas meter was 378300 cu. ft.; on Jan. 8 it was 376100 cu. ft. The bill was computed at 95¢ per 1000 cu. ft., with 10% off for prompt payment. What was the saving if the bill was paid promptly?

9. When the gross electric light bill was \$3.18, by prompt payment, \$3.02 settled the bill. What was the % of saving?

10. Those who have studied the problem carefully have estimated that $\frac{1}{2}$ gallon of gasoline is wasted daily by the owner of each of 6,000,000 passenger cars in the United States through useless mileage, overuse of cars, and carelessness. One gallon of gasoline provides the energy to move 15 tons of freight one mile. The wasted gasoline is equal to the freight haul of how many tons for one mile?

CHAPTER XII

INVESTMENTS

While a person is accumulating savings, he should be studying the problem of how his savings may be made to earn more money; in other words, he should be learning how to invest money. To do this wisely one must know about investments. It is the purpose of this chapter to give you some knowledge of investments, how to judge them correctly, and how to make them wisely.

A perfectly safe investment usually earns a low rate of interest. Do you see why? War Savings Stamps and United States Government bonds belong to this class.

A good investment yields a higher rate of interest than the class just mentioned, but the element of risk is somewhat greater. The bonds of cities, counties, and states; district drainage bonds; and promissory notes secured by mortgage, or by deed of trust, are examples of this type of investment.

Investments which offer a high rate of interest (say 10% or 12%) are to be regarded with suspicion by the person with small savings. Examples of this type are oil stocks and mining stocks. Transactions in such stocks should more properly be called speculations.

In general a high rate of interest usually means a large risk; a low rate of interest means a small risk. A first class investment can also be readily converted into cash in case of emergency.

Judging Investments

Which of the following investment advertisements do you think offers the safest place for your money? Which the next safest? Which the least safe? Why?

1. "Thrift is Power." "Save and Succeed."

Keep on Buying War Savings Stamps.

Series of 1920

On January 1, 1925, the United States Government will pay \$5.00 for each War Savings Stamp sold in 1920 at the following prices: Jan., \$4.12; Feb., \$4.13; March, \$4.14; April, \$4.15; etc.

2. "Nothing Venture, Nothing Gain."

Oil has made many fortunes over night, why not yours?

The Money-back Oil Co.

Stock now only 10¢ a share. Get in early before the price advances.

3. "Bonds as Safe as Our Cities."

Municipal bonds yielding 4% to 5% fulfill these tests for successful investing. These bonds are

(1) Safe.—Payment of principal and interest are assured when they are due.

(2) Marketable.—They can be readily sold at any time.

(3) Productive.—The interest yield is as high as is consistent with safety.

4. *Individual loans* secured by residences, apartments, and small business properties in Chicago, in amounts from \$2000 to \$20,000. Interest yield, 5% to 6%.

5. 6 percent \$500 and \$1000 denomination 1st Mortgage Serial Gold notes. Interest payable semi-annually. Secured by First Mortgage on Cleveland, Ohio, real estate of more than double value.

Consult the newspapers and magazines for investment opportunities and bring to class what you think are examples of poor, good, and excellent investments.

Mortgages on Real Estate

The oldest investment in the world is the first mortgage loan on land. The Babylonians invented mortgages more than 2500 years ago. Clay tablets unearthed and translated by scholars show that the mortgage in all essentials was the same then that it is now:

(1) A definite promise to pay at a designated time and place a certain sum of money bearing a specified rate of interest.

(2) A definite pledge of real estate as security to guarantee prompt payment of principal and interest.

Much real estate in the United States is pledged as security for mortgage loans. It was recently estimated by a committee of the American Bankers Association that more than \$6,000,000,000 in mortgages are held by savings banks and insurance companies in this country, and a total of \$10,000,000,000 is held by institutions and individuals as investments.

Mortgages are popular investments because as a class they yield a higher rate of interest than other investments with the same element of risk. Also in time of financial panic they are free from wide fluctuation in value.

Safety factors to be observed when negotiating a first-mortgage loan.

1. The loan should not exceed 50% of the cash value of the real estate pledged as security.

2. If buildings are part of the security, they should be insured in favor of the owner of the mortgage.

3. All back taxes against the property should be paid.

4. The property should be free from debt.

5. The mortgage should be promptly offered to the proper official for record.

Problems about Mortgage Loans

1. A man owned a \$2700 mortgage, bearing 6% interest, on a house on Lafayette Avenue; a \$3000 5% mortgage on Olive Street; and an \$8000 mortgage, earning $5\frac{1}{2}\%$ interest on Julian Avenue. How much was his average monthly interest income on these three investments?

2. When James Stewart was 21 years old, he had saved \$500. He lent this sum for five years at 6% interest per annum, payable annually, to John Smith, a young farmer, who owned 60 acres of land, free from debt, valued at \$4000. Mr. Smith gave as security for his note a mortgage on this land.

- (1) Who was the borrower in this transaction?
- (2) Who was the lender? Who signed the mortgage?
- (3) Who kept the note? Who kept the mortgage?
- (4) What was the face of the note?
- (5) When was the interest due? How much?
- (6) Write the note described in the problem.
- (7) Do you think this was a good investment for Mr.

Stewart? Why?

3. After owning for 3 months the note described in problem 2, Mr. Stewart sold it to Levi Miller for \$540.

(1) How much of the selling price of the note was earned interest? How much was premium?

(2) When did Mr. Miller get his first interest?

(3) Did Mr. Miller realize 6% on his investment? Why?

(4) What did Stewart do before Miller could become the legal owner of the note?

You have probably observed in your newspaper and magazine reading that the business of manufacturing, mining, shipping, and merchandising is carried on chiefly by companies or corporations. Find the names of several companies (1) in your community, (2) in your magazine or newspaper.

Corporations are authorized by the state to transact business just as individuals. The rights, duties, and privileges are stated in a document, called a **charter**, which is granted by the state to the corporation.

The members of the corporation elect from their number several persons (often five), called a **board of directors**. This body transacts the important business of the corporation. The less important business is transacted by the officers of the board, usually consisting of a president, a secretary, and a treasurer.

The capital used in the business is called **stock**. The owners of the stock are called the **stockholders**. The stock is divided into a certain number of equal parts, called **shares**. The face value of a share, called par value, is usually \$100, although mining and oil companies often issue shares of \$1 each. (Do you see any reason why?)

If the capital stock is \$25,000, divided into shares of \$100 each, there are 250 shares.

The instrument which is given the owner of stock is called a **stock certificate**, or **stock**. Study the form on the next page and find out how such a certificate differs from a promissory note.

The amount of money to be distributed from the earnings of a company among the owners of common stock is

Capital Stock \$100,000.00		1,000 Shares of \$100.00 each
The Acme Manufacturing Company		
Stock Certificate No. <u>2</u>		
This Certifies that <u>A. M. Miller</u>		
is entitled to <u>200</u> shares of One Hundred Dollars each, in the capital stock of the Acme Manufacturing Company, fully paid and non-assessable, transferable only on the books of the Corporation by the holder hereof in person or by attorney upon surrender of this certificate properly endorsed.		
In Witness whereof, the said corporation has caused this certificate to be signed by its duly authorized officers, and to be sealed with the Seal of the Corporation, at <u>Chicago, Ill.</u> this <u>27th</u> day of <u>May</u> A. D. 19 <u>—</u>		
	<u>E. J. Barber</u> Secretary and Treasurer.	<u>J. E. Colby</u> President

CERTIFICATE OF STOCK

determined by the board of directors. The money so distributed is called a **dividend**. The **rate of dividend** is always computed on the par value. Thus a 6% dividend means 6% of the par value, whether this be \$100, \$50, or \$1 per share.

The **market value** of a stock is the price it brings as a purchase, or as a sale. If a man pays \$80 for a \$100 share, the market value is \$80. This value may fluctuate from day to day or even during the same day. Newspapers always quote the market value.

Companies sometimes issue two kinds of stock, called **common** and **preferred**. Preferred stock has the dividend rate fixed in the stock certificate. This dividend must be set aside from the earnings of the company before a dividend on the common stock can be distributed.

Brokerage

A broker is an agent who buys or sells for the owners. The compensation which he receives is called brokerage. This is usually a certain sum for each share of stock bought or sold.

It is the custom of the New York Stock Exchange to allow the broker 20¢ per share for buying or selling any stock which brings \$125 or more per share, 15¢ per share for stock whose market value is between \$10 and \$125 per share, and $7\frac{1}{2}$ ¢ if the market value is less than \$10. (1924.)

Thus, the brokerage for selling 100 shares of Mo. Pacific R. R. stock (par value \$100) at $\$25\frac{7}{8}$ per share is $100 \times 15\text{¢}$, and for selling 100 shares of M. K. & T. R. R. stock (par value \$100) at $\$9\frac{1}{4}$ per share is $100 \times 7\frac{1}{2}\text{¢}$.

When brokerage is expressed in per cent it is computed on the par value.

Problems about Stocks

1. On a certain stock certificate is found this statement: "Capital 1000 shares of \$100 each." Express the capital in dollars.

2. How many shares of \$100 each in a company's capital of \$1,000,000?

3. A man owns 10 shares of bank stock. On his stock certificate is the statement: "Capital \$60000; shares \$100 each." What % of the capital stock does he own?

4. How much must you pay, including brokerage, for 10 shares of United Railway stock when it is quoted at \$8.25?

5. How much in money is a 6% dividend when the par value is \$100? \$50? \$1?

6. What is the probable effect of a high dividend rate on the market value? Why?

How to Read the Stock Market

Table I shows the prices of certain stocks during the day, the change over the preceding day, the last quotations at the close, the net change, and the number of shares sold on the Stock Exchange at New York City on Aug. 5, 1918.

TABLE I

Sales	Name of Stock	Prev. Close	Open	High	Low	Close	Net Change
400	Balt. and O. . .	54	$54\frac{1}{8}$	$54\frac{3}{8}$	54	54	0
200	do* preferred	57	57	57	57	57	0
200	Colo. Fuel & Iron	45	$45\frac{1}{8}$	$45\frac{1}{8}$	45	45	0
500	Mo. Pacific . .	$23\frac{3}{8}$	$23\frac{1}{4}$	$23\frac{3}{8}$	$23\frac{1}{8}$	$23\frac{1}{4}$	$-\frac{1}{4}$
300	Norfolk & Western	103	103	$103\frac{1}{4}$	103	$103\frac{1}{4}$	$+\frac{1}{4}$
600	Wabash	$9\frac{1}{4}$	$9\frac{1}{2}$	$9\frac{1}{2}$	$9\frac{1}{4}$	$9\frac{1}{4}$	0
100	Woolworth . .	$112\frac{1}{2}$	111	111	111	111	$-1\frac{1}{2}$

The first line of this table is to be read as follows: On August 5, 1918, there were sold on the New York Stock Exchange 400 shares of common stock of Baltimore and Ohio Railroad. The first sale (called "Open" in the table) was made at $\$54\frac{1}{8}$ per share; the highest price was $\$54\frac{3}{8}$; the lowest price was \$54, which was also the price of the last sale of the day (called "Close" in the table). Net change means the difference in price between the last sale of the day and the last sale of the previous day. In Baltimore and Ohio there was no net change, although the price fluctuated during the day.

*The second line means this: There were sold 200 shares of Baltimore and Ohio preferred stock (*do means the same) at \$57 per share, there being neither net change nor fluctuation during the day.

Tell the meaning of the rest of Table I.

STOCK QUOTATIONS

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Table II shows the sales and prices on Dec. 24, 1919, of the stocks listed in Table I. It is to be interpreted exactly as Table I was interpreted.

TABLE II

Sales	Name of Stock	Prev. Close	Open	High	Low	Close	Net Change
6400	Balt. and Ohio.	32	32	$32\frac{5}{8}$	$31\frac{3}{4}$	$32\frac{1}{4}$	?
1200	do preferred	$46\frac{1}{2}$	46	46	$45\frac{1}{4}$	$45\frac{1}{4}$	?
200	Colo. Fuel & Iron	40	$39\frac{5}{8}$	40	$39\frac{5}{8}$	40	?
1000	Mo. Pacific	$25\frac{3}{4}$	$25\frac{5}{8}$	$25\frac{7}{8}$	$25\frac{1}{2}$	$25\frac{3}{4}$	?
1400	Wabash	$8\frac{1}{8}$	$8\frac{1}{8}$	$8\frac{3}{8}$	8	8	?
100	Woolworth	$121\frac{3}{8}$	122	122	122	122	?

1. Find the amount of net change for each stock.
2. Tell the meaning of Table II for each stock.
3. Look on the market page of your newspaper for the quotations on the stocks listed in this table.

Problems Based on Tables I and II

1. Name the stocks in Table I which sold at a discount (below par). Name those which sold at a premium (above par).
2. Which stock sold at a premium in Table I and is not listed in Table II?
3. Name the stocks which made a plus net change in Table II.
4. Find the total sales in shares in Table I.
5. Compute the ratio between the total shares sold in Table II and those sold in Table I.
6. A man invested \$5000 in Missouri Pacific Railroad stock at $25\frac{5}{8}$, plus 15¢ brokerage. How many shares did he buy and how much money had he left?

7. How much money was sent to the owner by the broker who sold 1000 shares of Norfolk and Western Railroad stock at $98\frac{1}{4}$?

8. What was the total cost to the man who bought 200 shares of Balt. and O. stock at 54 on Aug. 5, 1918, plus brokerage at 15¢ per share?

9. If the buyer in problem 8 sold his stock on Dec. 24, 1919, at $32\frac{5}{8}$, less brokerage, find his total loss.

10. If Balt. and O. stock paid no dividend for 1918 and 1919, what additional loss did the owner of such stock sustain?

11. Compute the loss of the man who bought 300 shares of Wabash at $9\frac{1}{2}$ and sold it at $8\frac{3}{8}$, brokerage $7\frac{1}{2}$ ¢ per share in each transaction.

12. Compute the gain of the owner of 100 shares of Woolworth stock if it was bought at its lowest price in Table I and sold at the highest price in Table II. Do not omit brokerage.

13. Some persons think that stock transactions should be classed as speculations rather than as investments. Do you now see why they think so?

14. Select five stocks from your daily paper and keep a daily record for a week of the net change of each. At the close of the week express your results in 5 suitable graphs.

How to Compute the Rate of Income

In the case of stocks the amount of the dividend on each share is also the income on a share.

The dividend = par value $\times$ rate of dividend.

The rate of income is the per cent which the income is of the purchase price (market value at the time the stock was bought).

This formula is helpful in solving certain stock problems.

$\text{Par value} \times \text{rate of dividend} = \text{market value} \times \text{rate of income}.$

PROBLEM.—When 6% stock is selling at 120, find the rate of income. \$6 is the income produced by each share. To get this income the buyer paid \$120. The problem then becomes an easy one in percentage. “\$6 is how many per cent of \$120?”

PROOF.— $\$100 \text{ (Par)} \times 6\% \text{ (Rate of dividend)} = \$120 \text{ (Market)} \times 5\% \text{ (Rate of income)}.$

The formula also helps to solve this type of problem: “What may be paid for a 6% stock to make 8% on the purchase price (market value)?”

SOLUTION.—A 6% stock yields \$6 income on each share. The problem then becomes “\$6 is 8% of what amount?” \$6 is 8% of \$75 (market value).

PROOF.— $\$100 \text{ (Par)} \times 6\% \text{ (Rate of dividend)} = \$75 \text{ (Market)} \times 8\% \text{ (Rate of income)}.$

Problems

1. When U. S. Steel 7% stock was selling at 105, was the rate of income more or less than 7%? Why?

2. When 4% railroad stock sold at 85, was the rate of income as much as 5%? At what price must this stock be bought to earn exactly 5% income?

3. When a certain bank stock was selling at 115, its dividend rate was 10%. Find the rate of income.

4. In 1920, the bank stock mentioned in problem 3 sold at 150. What was the rate of income at this price?

5. If a buyer is satisfied with a 5% income on a 10% bank stock, what can he afford to pay for the stock?

INVESTMENTS

COUPON BOND

UNITED STATES OF AMERICA

FIRST MORTGAGE 5 7/8 SINKING FUND GOLD BOND.

OF THE
CITIZENS STREET RAILWAY COMPANY,
INCORPORATED UNDER THE LAWS OF MICHIGAN.
TOTAL AUTHORIZED ISSUE \$250,000.

To Have Received: The Citizens Street Railway Company established in the City of
Hialemaize in the County of Hialemaize and State of Michigan promises to pay, there-
after, or if registered, to the registered holder, the sum of One Thousand Dollars in
gold coin of the United States of the present standard of weight and fineness at the office
of the Illinois Trust and Savings Bank, Trustee in the city of Chicago, Illinois, on Twenty
Five Years from the date hereof, or at will, at or after the expiration of five years from date,
with specified premium, as hereinafter provided; both interest on said sum at the rate of
six per cent per annum, payable semi-annually, on like gold coin, on the first days of
November in each year, at the office of said Illinois Trust and Savings
Bank, in the city of Chicago, Illinois, subject to the payment of the coupons hereon attached,
when the same have been taken and surrendered herefor respectively.

This Bond is one of a series of Two hundred and fifty bonds for One
Thousand Dollars each, numbered and designated by Serial Number and by Sinking Fund
Debt Certificate and certificate serially from 1 to 250, issued on date hereof, and
payable at the term hereof, and by the same term of Twenty Five Years secured and delivered in
full payment of the Bonds dated and issued at Hialemaize, Illinois, in full payment of the
Bonds of said company authorizing the issue of said bonds and the execution and delivery of the
Trust Deed hereinafter mentioned.

Payment of this bond and of the coupons attached is secured by a Trust
Deed of even date herewith (which is hereby referred to and made a part of this bond) now
having all the property, assets and franchises (both present and future acquired) of said
Citizens Street Railway Company, executed and delivered by it to said Trustee.
Said Trust Deed (which has been duly recorded and filed both as a mortgage on real estate
and as a chattel mortgage), is in the possession of said Trustee, open to examination
by any party having any lawful claim.

This bond may at the option of the holder be redeemed at or after the expira-
tion of five years from the date hereof and before maturity by payment in gold
coin or equivalent of one thousand and twenty dollars (\$1020) together with
all interest accrued to the time of such payment and unpaid, plus so far as
will discharge having first been given by public sale and acquiesced by the terms of said
Trust Deed.

This bond shall not be valid unless the certificate endorsed hereon shall bear the
signature of the Trustee.

In Testimony Whereof said Citizens Street Railway Company has caused this certificate to
be signed in its presence and its corporate seal to be hereunto put and attested by
its Secretary and the official copies are to be authenticated by its corporate seal, which
it has caused to appear this first day of November A.D. Eighteen Hundred and Ninety Three.

Citizens Street Railway Company

Witness my hand and seal of office this _____ day of _____ 18____.

Secretary

NOTE.—The above form shows only the last ten of the fifty coupons originally attached. This is a 25-year bond with two interest coupons falling due each year.

Bonds

1. National, state, county, and city government, and other corporations sometimes borrow money to meet certain expenditures. This may be done by selling bonds.

2. A bond is a promise under seal by a corporation to pay a certain sum of money at a specified time.

3. Study the picture of the coupon bond on page 350 to discover (1) in how many ways it is like an ordinary promissory note; (2) in how many ways it is different.

4. Write the resemblances and differences.

5. Bonds may be issued in any denomination, but the usual size is \$1000, \$500, or \$100.

6. Bonds usually have a long time to run, such as 15, 20, or 30 years.

7. Turn to page 344, and read carefully once more a stock certificate.

8. Write at least three important ways in which the bond on page 350 differs from the stock certificate just read.

9. Some pupils find five important differences. Can you?

10. Study the interest note (coupon) on this page. Then answer these questions.

(1) How often is the interest due?

(2) What is the face of the bond from which this coupon was cut?

(3) If you were the owner of this coupon, how would you get payment for it? When?

(4) What care must be taken of coupon bonds?

THE UNITED STATES OF AMERICA will pay to bearer on March 15, 1922, at the Treasury Department, Washington, or at a designated agency, \$1.06, being six months' interest then due on a $4\frac{1}{4}\%$ \$50 Coupon Bond of the Third Liberty Loan. 1379535

Register of the Treasury.

Buying Bonds

1. Bonds are bought and sold through brokers. Your banker can also obtain them for you.

2. Why may it be better for the small investor to buy his bonds from his banker?

3. Young investors should early make the acquaintance of the local banker. Why?

4. The brokerage, premium, and discount are computed on the par value (face value).

5. Interest on bonds is due at intervals named in the bond.

6. When bonds are bought between interest dates, the buyer pays the interest earned to the day of purchase. This is called **accrued interest** and it comes back to the owner of the bonds at the next payment of interest.

7. On Nov. 27, 1920, $4\frac{1}{4}\%$ Third Liberty Loan bonds were quoted at \$88.50. This means that the buyer must pay \$88.50 for a \$100 bond, also the interest on \$100 at $4\frac{1}{4}\%$ from Sept. 15, 1920, to Nov. 27.

8. If you count the actual days, how much is the accrued interest in problem 7? Why is it right for the buyer to pay this accrued interest?

9. How much money did I pay on Nov. 27 for three \$100 bonds based on the quotation in problem 7, omitting the brokerage?

10. Find the cost of ten one thousand dollar U. S. 2's when quoted at 98, allowing $\frac{1}{8}\%$ brokerage, if bought immediately after the interest is due.

11. Make and solve five problems based on the price of Liberty Loan bonds at the time you are studying this page.

12. On Dec. 24, 1919, $4\frac{1}{4}\%$ Fourth Liberty Loan bonds were quoted at \$91.40. The last interest payment on these bonds was Oct. 15, 1919. How many \$100 bonds, brokerage $\frac{1}{8}\%$, could have been bought with \$5050 and how much money was left?

HINT.—Remember that the total amount paid for one bond is a sum of three addends.

13. With the money remaining in problem 12, how many \$50 Fourth Liberty Loan bonds could have been bought on Dec. 24? How much money remained? How might the money left have been safely invested?

14. Consult a newspaper which contains the bond market and find out how many \$50 Liberty Loan bonds described in problems 12 and 13 you can buy for \$5000 at the time you study bonds at school.

Finding the rate of interest bonds earn for the buyer, assuming that the bonds are held until maturity and that the face value is paid at that time.

1. The bond interest is the interest **named** in the bond.

2. The buyer's interest is the interest **earned** by the bond.

It may be equal to, less, or greater than the bond interest, accordingly as the bond is purchased at par, at a premium, or at a discount.

3. If a 5% \$100 bond, due in 6 years, is selling at 104, the total bond interest for 6 yr. is $6 \times \$5$, whereas the interest (income) actually earned by the investment is \$30 less \$4 (premium), or \$26, because the bond at maturity is paid at par value.

4. If a 5% \$100 bond due in 3 yr. is bought at 98, the total bond interest is $3 \times \$5$, but the total interest (income) actually earned is \$15 and \$2 (discount), or \$17.

5. The rate of interest a bond earns for its buyer is based on the amount paid for the bond, exclusive of accrued interest which is returned to the owner at the time he receives his first interest.

6. In problem 3 the actual interest earned in 6 yr. by an investment of \$104 is \$26. The interest earned in 1 yr. is $\$26 \div 6$, or $\$4\frac{1}{3}$. Consequently the rate of interest the buyer actually gets is $\$4\frac{1}{3} \div \$104 = 4.16 + \%$.

7. If the reasoning in problem 6 is employed on the bond in problem 4, the rate of interest to the buyer = $\frac{\$17 \div 3}{\$98}$
 $= \frac{\$5\frac{2}{3}}{\$98} = 5.78 + \%$.

8. The next formula will be found convenient in solving bond problems.

Par value + Bond interest = Market value + Buyer's interest.

9. What rate of interest do 6% bonds yield, due in 7 years, if bought at 108?

$\$100 + \$42 = \$108 + \$x$ (Buyer's interest). See formula in number 8.
 $\$x = \$34$.

Rate of interest yielded to buyer = $\frac{\$34 \div 7}{\$108} = 4\frac{1}{2}\%$ nearly.

NOTE.—In the above computation simple interest is used. Bankers use tables in which the principle of compound interest is employed. On bonds with a short time to run, the difference between the two methods is very small. For all practical purposes the ordinary bond buyer may use the method indicated above.

10. When Third Liberty Loan $4\frac{1}{4}\%$ bonds had exactly 9 years to run until due, they were quoted at 93.20. Omitting brokerage, find the buyer's rate of interest.

11. If 6% bonds selling at 112 have exactly $15\frac{1}{2}$ years to run, determine the buyer's rate of interest.

12. When U. S. 4's were selling at 106, 7 years and 3 months before maturity, what was the rate of interest to the buyer?

13. On Nov. 29, 1919, a United States Treasury certificate, worth \$100 on Jan. 1, 1924, was purchased for \$84.40. Some people said that at this price the investment was earning 4.27% compound interest, compounded yearly; others said the rate of interest earned was only $4\frac{1}{4}\%$ compounded yearly. Use your knowledge of compound interest to see which statement is correct.

14. By use of the compound interest table on page 320, determine whether a \$1000 United States Treasury certificate due Jan. 1, 1925, purchased Jan. 1, 1920, for \$824, earns as much as 4% interest, compounded yearly.

15. What rate of simple interest are Third Liberty Loan bonds earning the buyer on the day you are studying this page?

16. Compute the rate of interest earned by Fourth Liberty Loan bonds if they are bought at the current price.

Why We Should Want to Own Good Bonds

1. United States Government and many municipal bonds are the safest investment that can be made. There may be others just as safe, but none safer.

2. The interest is paid promptly at the time it is due.
3. They can be bought in small denominations.
4. They are good security at banks if the owner needs to borrow money for a short time.
5. There is always a cash market for them. This is advantageous both to buyer and seller.
6. In the case of registered bonds the interest is **sent** to the owner.
7. To the person with a small income, United States bonds are free from all tax.
8. Their depreciation (loss in market value) is likely to be less than that of most other property.

How Bonds and Stocks Differ

1. Bonds are promises to pay a certain sum of money at a definite time. Stock certificates are merely evidence of ownership.
2. The amount of interest and the time of its payment are definitely fixed. Dividends are not certain.
3. The payment of bonds is guaranteed by some form of security. There is no guarantee of any kind in a stock.

Why Persons with a Small Income Should not Buy Stocks for Investment

1. The market value of most stocks fluctuates widely.
2. Dividends are not certain.
3. The rate of dividend is not fixed except in preferred stocks. Even in this case it is not guaranteed.
4. The speculative factor is too large.

1. A farmer bought a horse on Apr. 1 for \$175. On Oct. 1 he sold him for \$200. If the work done by the horse paid for his board, and if money was worth 6% to the farmer, find the per cent of profit on the selling price.

2. In 1918 a certain level Ohio farm averaged 23 bu. of wheat per acre, valued at \$2.15 per bu. The following items show the acre cost of growing wheat on this farm:

Plowing, 7 hr. @ 60¢;

Harrowing and disking, 5 hr. @ 60¢;

Seed, 2 bu. @ \$2.15; fertilizer, 250 lb. @ \$20 per ton;

Interest on the above labor and expense at 6%, \$.84;

Interest on 1 acre of land valued at \$150 at 6%;

Harvesting and threshing, \$3.30;

Delivering to market 1 mile away, \$.60.

Determine the profit per acre on this farm. Is growing wheat a profitable farm practice under these conditions? Is there any item of cost omitted in the statement above?

3. If the farm were hilly and located 7 miles from market, how would the cost items vary in the previous problem?

4. A certain Ohio farmer in 1918 raised 5 acres of potatoes averaging 110 bushels per acre. He sold them at \$1.25 per bushel. It required 14 bushels of seed potatoes valued at \$1.50 per bushel for planting an acre. Materials for spraying cost \$1.60 per acre. The labor per acre was estimated at 45 hr. (man and team) at 75¢ per hr. The cost of picking up and sacking was \$2.25 per acre. What was the gross profit per acre? What was the farmer's net profit on 5 acres after deducting interest @ 6% on land valued at \$120 per acre, also interest @ 6% for 3 mo. on his labor and expense, and a $1\frac{1}{2}\%$ tax on a tax valuation of \$90 per acre?

Problems About Real Estate

1. If you have \$7200 to invest, which will be the better proposition, lending it at 6% interest, or buying a small business property which rents for \$55 a month? Allow \$125 a year for repairs, \$40 for insurance, and \$70 for taxes.

2. A man bought for \$7000 a storehouse, which he rented for \$40.00 a month. He paid annually \$65 for repairs, \$50.00 for taxes, and \$35.00 for insurance. At the end of two years he sold the property for \$7500. Did he make more, or less, than he would have made by putting his money into a savings bank which pays 4% interest, compounded semi-annually?

3. I paid \$4000 for a house and lot. The taxes are \$45 per year, and other yearly expenses are estimated at \$60. What rent must I charge per month to make 6% interest on the cost of my property, if the house is vacant on an average $8\frac{1}{3}\%$ of the time?

4. A man paying \$60 a month house rent has \$4000 in a savings bank. The bank pays him 4% interest per annum, compounded semi-annually. Will he save or lose money by buying a house for \$10,000 with the \$4000 cash and a 6% note for the balance, if taxes on the house are \$115 a year and repairs and other expenses are \$100? How much?

5. James Forest has \$1250. He can lend it to Alfred Wood for one year at 6%, or he can purchase with it a farm mortgage due in 3 yr. with interest at $5\frac{1}{2}\%$ per annum. What would be Mr. Forest's income in each case from his investment? State in writing all the reasons you can think of why he should prefer the farm mortgage.

1. In Texas the legal rate of interest is 8%. In Ohio it is 6%. How much greater is the Texas rate than the Ohio rate? What per cent greater?

2. In the window of a bank of a large city is found this statement, "We pay 4% per annum on your savings." Across the street in another bank's window is found this statement, "We pay 25% more." What does this mean?

3. A young man on July 10, 1912, bought 10 shares of bank stock (par value \$100) at \$115 per share, no brokerage. This stock paid a 10% dividend each year, $\frac{1}{2}$ paid Jan. 1 and $\frac{1}{2}$ paid July 1. He owned this stock Jan. 15, 1924.

(1) What rate of income was realized by the owner?

(2) What amount in dividends was paid to the owner?

(3) The owner of this stock placed each dividend, as soon as it was received, into the savings bank at 4% simple interest. How much had the dividends earned for him in this manner on Jan. 1, 1924.

HINT.—The first dividend, \$50, was deposited Jan. 2, 1913. The second was deposited before July 5, 1913. Each dividend was deposited on or before the 4th of the month in which it was due.

(4) On Jan. 20, 1924, this stock was sold at \$150 per share. What was the rate of profit?

(5) What was the rate of income to the new owner?

4. In July, 1899, a teacher bought a \$1000 5% Florence, Ala., bond, interest payable semi-annually. How much interest had this investment earned for the owner when the bond was paid, Jan., 1919, if the owner re-invested each interest coupon of \$25.00 at 4% the day it was due?

Investing in a High School Education

The Federal Bureau of Education at Washington has found out that boys leaving school at 14 receive an average weekly wage of \$4; at 18 these boys receive an average of \$7 a week; at 20 they get an average of \$9.50 per week; at 25 they get an average of \$12.75 per week. The average total wages at 25 for such boys are \$5,112.50.

The boys who stop school at 18 to begin work get on an average \$10 per week; at 20 they get \$15; and at 25 they are getting \$31. The average total earned for such boys at 25 is \$7,337.50.

1. Construct graphs showing the average weekly wage of the two groups at the ages named.

2. Find the ratio of the total wages at 25 of the boy who remained in school till 18 to the total wages of the one who quit at 14.

3. Was the additional time spent in school a good investment?

4. Which of these groups will first reach the limit of their earning power? Why?

5. What opportunities may the high school boy have at 25 years of age, or older, which probably would never come to the boy who quit school at 14?

6. Would it be a good investment for a young person without means to borrow from some friend \$100 a year, at 6% simple interest, for four years in order to go to high school?

CHAPTER XIII

REVIEWING THE YEAR'S WORK

Checking Addition

I

Check these results in 5 minutes.

1.	2.	3.	4.	5.	6.
3947	7892	4896	8390	3553	5377
8325	1040	2463	4099	6289	0292
6482	6238	2949	6253	2287	3462
9638	4892	6284	9246	3264	7385
4358	3622	4392	7839	5389	2462
2622	2363	6250	6284	2622	1963
8301	5248	9083	3392	5483	8424
5289	9099	5969	1564	9060	5689
6384	2262	7235	1763	5263	3562
7236	3104	6396	1888	7482	9842
5847	2042	5122	8881	9628	5620
2282	3969	2215	3089	3385	3356
<u>70711</u>	<u>51771</u>	<u>63254</u>	<u>62688</u>	<u>63705</u>	<u>64434</u>

II

The account below shows for a week the daily and total collections of each of six cashiers for a certain gas company and the commission due each cashier at 1%.

There are several errors in the totals. Find them and check your work. How may you know that your work is right?

Cashier	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total	Cashier's Pay
Miss Jones	\$ 418.20	\$ 392.16	\$ 408.95	\$ 375.62	\$ 296.90	\$ 425.16	\$ 2316.99	\$ 23.17
Miss Will	428.40	250.20	317.75	296.18	324.82	386.24	2003.50	20.04
Miss Arnold	432.15	252.65	400.20	385.62	216.94	465.70	2163.26	21.63
Miss Casey	296.33	384.24	253.16	484.20	416.10	258.95	2092.98	20.93
Miss Baker	304.15	215.93	392.99	456.32	389.05	304.01	2062.45	20.62
Miss Sands	286.66	310.04	464.82	562.80	362.19	218.92	2200.43	22.00
Totals	\$2160.89	\$1804.22	\$2237.87	\$2560.74	\$2006.00	\$2058.98	\$12834.70	\$128.31

Checking a Subtraction Test

The next work is a copy of a junior high pupil's paper on a subtraction test. Each correct answer is worth $3\frac{1}{3}$. The paper was marked 100. Find out if this was the right grade. If your answer is "No," find the right grade.

	a	b	c	d	e
1.	$\begin{array}{r} 5263 \\ 1825 \\ \hline 3438 \end{array}$	$\begin{array}{r} 6352 \\ 4918 \\ \hline 1434 \end{array}$	$\begin{array}{r} 1546 \\ 1029 \\ \hline 517 \end{array}$	$\begin{array}{r} 6904 \\ 3258 \\ \hline 3646 \end{array}$	$\begin{array}{r} 5362 \\ 4284 \\ \hline 1078 \end{array}$
2.	$\begin{array}{r} 2784 \\ 1279 \\ \hline 1505 \end{array}$	$\begin{array}{r} 5248 \\ 2967 \\ \hline 2281 \end{array}$	$\begin{array}{r} 1900 \\ 1375 \\ \hline 525 \end{array}$	$\begin{array}{r} 5426 \\ 2879 \\ \hline 2547 \end{array}$	$\begin{array}{r} 3000 \\ 2148 \\ \hline 852 \end{array}$
3.	$\begin{array}{r} 9000 \\ 2462 \\ \hline 6538 \end{array}$	$\begin{array}{r} 3642 \\ 2984 \\ \hline 658 \end{array}$	$\begin{array}{r} 6397 \\ 2989 \\ \hline 3408 \end{array}$	$\begin{array}{r} 6014 \\ 5295 \\ \hline 719 \end{array}$	$\begin{array}{r} 3021 \\ 2674 \\ \hline 347 \end{array}$
4.	$\begin{array}{r} 3048 \\ 2659 \\ \hline 389 \end{array}$	$\begin{array}{r} 2863 \\ 1678 \\ \hline 1185 \end{array}$	$\begin{array}{r} 6072 \\ 5994 \\ \hline 78 \end{array}$	$\begin{array}{r} 4321 \\ 1895 \\ \hline 2426 \end{array}$	$\begin{array}{r} 2601 \\ 1452 \\ \hline 1149 \end{array}$
5.	$\begin{array}{r} 3982 \\ 2946 \\ \hline 1036 \end{array}$	$\begin{array}{r} 5343 \\ 3435 \\ \hline 1908 \end{array}$	$\begin{array}{r} 1684 \\ 1596 \\ \hline 78 \end{array}$	$\begin{array}{r} 7000 \\ 2482 \\ \hline 4518 \end{array}$	$\begin{array}{r} 5000 \\ 3604 \\ \hline 1396 \end{array}$
6.	$\begin{array}{r} 8742 \\ 3695 \\ \hline 5047 \end{array}$	$\begin{array}{r} 8004 \\ 2995 \\ \hline 5009 \end{array}$	$\begin{array}{r} 3009 \\ 1998 \\ \hline 1011 \end{array}$	$\begin{array}{r} 63780 \\ 45999 \\ \hline 16781 \end{array}$	$\begin{array}{r} 7104 \\ 6908 \\ \hline 196 \end{array}$

Find the errors in these 3 minute multiplication tests.

I

a

1. $\frac{3}{4} \times 84 = 63$
2. $\frac{2}{3} \times 58 = 38\frac{1}{3}$
3. $\frac{2}{3} \times \frac{3}{24} = \frac{1}{12}$
4. $2\frac{1}{2} \times 2\frac{1}{2} = 6\frac{1}{4}$

b

1. $64 \times \frac{5}{8} = 40$
2. $3\frac{1}{2} \times 2\frac{3}{4} = 9\frac{5}{8}$
3. $\frac{1}{2} \times 12\frac{1}{2} = 6\frac{1}{4}$
4. $\frac{2}{3} \times 37\frac{1}{4} = 25$

c

1. $\frac{3}{8} \times \frac{5}{12} = \frac{5}{32}$
2. $\frac{2}{3} \times 7\frac{1}{2} = 5\frac{1}{2}$
3. $\frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$
4. $\frac{5}{12} \times 84\frac{1}{3} = 35\frac{5}{6}$

II

5624	9389	5158	3528	6809	9807
18	79	42	35	69	91
<u>101232</u>	<u>741731</u>	<u>216636</u>	<u>123480</u>	<u>479821</u>	<u>892437</u>

III

43957	52468	73098	87692	30296
73	46	29	37	31
<u>3208861</u>	<u>2413528</u>	<u>2119842</u>	<u>3244604</u>	<u>930176</u>

IV

45239	53429	60871	39245
278	951	273	436
<u>12576442</u>	<u>50810979</u>	<u>16617783</u>	<u>17110820</u>

V

406	550	640	802	302	340
22	36	60%	40	204	560
<u>8932</u>	<u>19900</u>	<u>384</u>	<u>32080</u>	<u>61608</u>	<u>190400</u>

VI

424	35.8	6.98	25.4	8.75	.256
.36	.42	64	.035	.028	.74
<u>15.264</u>	<u>15.036</u>	<u>446.72</u>	<u>.889</u>	<u>.345</u>	<u>.18944</u>

Checking Division Tests

Find the wrong answers and correct them in these 3 minute division tests.

I

a

- $\frac{3}{4} \div \frac{1}{4} = 3$
- $\frac{3}{4} \div \frac{5}{8} = 1\frac{1}{5}$
- $\frac{3}{8} \div \frac{3}{16} = 2$
- $\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2}$

b

- $\frac{1}{2} \div \frac{1}{4} = 2$
- $\frac{5}{12} \div \frac{1}{3} = 1\frac{1}{4}$
- $\frac{7}{12} \div \frac{7}{8} = \frac{2}{3}$
- $\frac{3}{8} \div \frac{3}{4} = 2$

c

- $\frac{3}{8} \div \frac{1}{4} = \frac{1}{2}$
- $\frac{7}{8} \div \frac{1}{2} = 1\frac{3}{4}$
- $\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4}$
- $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3}$

II

	a	b	c	d	e
1.	$\begin{array}{r} 1 \\ 12 \overline{)12} \end{array}$	$\begin{array}{r} .001 \\ 2.5 \overline{).025} \end{array}$	$\begin{array}{r} .01 \\ 12 \overline{).12} \end{array}$	$\begin{array}{r} .60 \\ .02 \overline{)12} \end{array}$	$\begin{array}{r} 10 \\ .001 \overline{).01} \end{array}$
2.	$\begin{array}{r} 100 \\ .1 \overline{)10} \end{array}$	$\begin{array}{r} 5 \\ .17 \overline{).85} \end{array}$	$\begin{array}{r} 300 \\ .25 \overline{).75} \end{array}$	$\begin{array}{r} .3 \\ .5 \overline{).15} \end{array}$	$\begin{array}{r} .1 \\ 840 \overline{)84} \end{array}$
3.	$\begin{array}{r} 10 \\ 8.4 \overline{)84} \end{array}$	$\begin{array}{r} 80 \\ .8 \overline{)6.4} \end{array}$	$\begin{array}{r} .04 \\ 21 \overline{).84} \end{array}$	$\begin{array}{r} 2 \\ .02 \overline{).004} \end{array}$	$\begin{array}{r} .04 \\ 81 \overline{)3.24} \end{array}$
4.	$\begin{array}{r} 40 \\ .021 \overline{).84} \end{array}$	$\begin{array}{r} .4 \\ .9 \overline{).36} \end{array}$	$\begin{array}{r} 10 \\ 3. \overline{)3} \end{array}$	$\begin{array}{r} 12 \\ 1.2 \overline{)14.4} \end{array}$	$\begin{array}{r} 1 \\ .3 \overline{).30} \end{array}$

III

	a	b	c
1.	$12 \div 12 = 1$	$.12 \div 12 = .01$	$.01 \div .001 = 10$
2.	$10 \div .1 = 100$	$75 \div 25 = 3$	$75 \div .25 = 300$
3.	$84 \div 840 = .1$	$84 \div 8.4 = 10$	$.84 \div 21 = .04$
4.	$3.24 \div 81 = .04$	$.84 \div .021 = 40$	$3 \div \frac{1}{3} = 9$
5.	$1.25 \div .05 = 2.5$	$.64 \div .08 = .8$	$6.3 \div 3 = 21$
6.	$.8 \div .25 = 3.2$	$3.6 \div 9 = .4$	$3 \div .0\frac{1}{3} = 90$

Find the error in these accounts.

Dr.		Cash	Cr.		
Mar. 1	To Balance.....	\$ 25.40	Mar. 5	By Rent.....	\$ 20.00
" 3	" Salary.....	90.00	" 9	" Shoes.....	5.00
" 7	" Interest.....	15.00	" 11	" Groceries.....	6.20
			" 17	" Clothing.....	25.00
			" 18	" Balance.....	72.60
		<u>\$130.00</u>			<u>\$130.40</u>

Dr.			Frank Fox	Cr.		
1923				1923		
Mar. 1	To Balance.....	\$ 15.60	Mar. 5	By Cash.....	\$ 12.00	
" 8	" Labor 5 da. @		" 12	" 6 bu. wheat		
	\$1.90.....	9.40		@ \$2.....	12.00	
" 15	" Labor 5½ da. @		" 17	" Cash for bal-		
	\$2.00.....	10.00		ance.....	9.20	

Dr.		Cash	Cr	
1923			1923	
Apr. 1	To Balance on hand	\$ 12.50	Apr. 3	By Clothing (shoes) \$ 4.50
" 6	To Wages (working after school and Sat.) . . .	8.00	" 8	" A m u s e m e n t (movies) 15
		<u>\$ 20.50</u>	" 11	" Church (contri- bution) 25
			" 12	" Balance 14.60
Apr. 12	To Balance on hand	\$ 14.60		<u>\$ 20.50</u>

A Speed Test in Checking Addition

Find out in 10 min. why this boy's paper was marked 75%.
Be sure to check your own answer if it does not agree with
the boy's answer which is printed below each example.

234	783	134	215	245	864
979	420	979	787	306	977
567	828	305	478	498	590
478	539	887	196	589	745
754	354	260	509	250	839
992	196	456	924	615	986
366	217	392	840	824	692
439	604	910	382	707	548
329	975	129	623	191	704
<u>5138</u>	<u>4916</u>	<u>4452</u>	<u>4960</u>	<u>4225</u>	<u>6945</u>

319	932	577	205	586	170
985	749	194	110	324	676
758	288	538	324	247	505
264	539	636	638	174	237
437	672	860	411	958	458
801	416	575	539	435	722
126	284	751	747	913	384
598	219	402	156	692	899
677	704	389	200	709	968
<u>4965</u>	<u>4803</u>	<u>4932</u>	<u>3330</u>	<u>5038</u>	<u>5018</u>

A Speed Test in Checking Division

Check this work in 8 min.

573	212	390	681
88) <u>50336</u>	72) <u>15264</u>	39) <u>15210</u>	43) <u>29283</u>
924	432	524	580
63) <u>58275</u>	82) <u>35424</u>	19) <u>10146</u>	27) <u>15660</u>

Checking Subtraction

A Speed Test

Check these answers in 4 min. If your result does not agree with the one here given, prove it.

92531839	122362975	87545006	181540147
<u>54232991</u>	<u>91392880</u>	<u>79306239</u>	<u>99620308</u>
38298848	30790095	8228767	81919829
15025346	105604633	116504717	114126502
<u>12009056</u>	<u>94372654</u>	<u>46423608</u>	<u>57107940</u>
3016290	11231979	70081109	57018562

Checking a Multiplication Test

This girl's paper was marked 80. Find out by solving all the examples whether the paper was marked properly. Each correct answer is worth 10.

Prove your answer if it does not agree with the one here given.

8742	2143	4079	6785	9809
<u>360</u>	<u>89</u>	<u>57</u>	<u>96</u>	<u>42</u>
3147120	191727	232503	651360	411978
9786	6739	4921	9524	6785
<u>45</u>	<u>19</u>	<u>73</u>	<u>58</u>	<u>24</u>
340370	128041	359233	552392	162840

Acquiring Skill in Computing Discounts

I. Single Discounts.

Read these results without the help of pencil if possible. Business men often use pencil and paper to check results which they obtain at a glance mentally.

a	b	c
1. \$60 less 20%	\$2 less 40%	90¢ less $\frac{1}{9}$
2. 84 less 25%	8 less .25	45¢ less $33\frac{1}{3}\%$
3. 90 less 10%	9 less $\frac{1}{3}$	60¢ less 20%
4. 24 less $\frac{1}{4}$	4 less 10%	50¢ less $\frac{1}{2}$
5. 32 less $\frac{1}{8}$	3 less 5%	80¢ less $\frac{1}{5}$

II. Successive Discounts.

Find the missing number. Use the single equivalent discount if it is helpful to do so. Use pencil only when necessary.

a			b		
List	Discounts	Net	List	Discounts	Net
1. \$40	20%, 10%	?	\$200	$33\frac{1}{3}\%$, 10%	?
2. 100	15%, 10%	?	300	$16\frac{2}{3}\%$, 10%	?
3. 80	10%, 5%	?	400	$\frac{1}{4}$, $\frac{1}{8}$	?
4. 50	$\frac{1}{2}$, $\frac{1}{4}$	?	250	10%, 5%	?
5. 20	$\frac{1}{4}$, 10%	?	800	20%, 10%, 5%	?
6. 70	30%, 20%	?	300	10%, 10%, 10%	?
7. 30	40%, 10%	?	480	$\frac{1}{4}$, $\frac{1}{6}$	?
8. 22	10%, 5%	?	500	10%, 2%	?
9. 75	20%, 5%	?	240	$\frac{1}{3}$, $\frac{1}{4}$, 5%	?
10. 10	25%, 10%	?	900	20%, 10%, 5%	?
11. 18	$\frac{1}{6}$, 10%	?	600	.25, .10, .05	?

Copy the example only if it is necessary.

Good eighth year pupils are not satisfied with anything less than 100% accuracy.

The following battery of inventory tests is designed to cover all the fundamental facts applied to the four fundamental processes.

These 45 examples contain the 100 addition facts if adding is begun at the bottom of the column.

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
2	5	2	7	6	5	1	7	3	7	6	9	4	6	5
3	0	0	8	5	9	8	1	5	4	8	6	1	0	2
4	1	9	8	1	7	0	4	2	0	9	3	2	4	3
5	7	0	0	4	1	1	1	1	0	0	1	4	3	2
8	9	6	8	7	7	6	3	5	7	5	5	8	7	8
5	7	2	2	2	0	5	3	4	9	7	9	4	9	5
1	6	3	2	6	5	1	0	0	1	9	8	7	6	9
1	1	0	0	0	1	3	4	2	0	2	2	2	3	4
7	5	7	8	5	8	9	7	7	7	9	0	2	6	7
9	9	9	7	8	5	6	2	4	0	8	9	6	6	3
5	7	0	1	1	2	4	3	6	1	2	6	7	1	6
0	0	3	8	2	7	4	5	2	2	3	0	5	5	0

Subtraction**Set A**

These examples contain 90 subtraction facts, each occurring but once, by the Method of Equal Additions.

7598719 <u>4887916</u>	76697666 <u>22416569</u>	9562145 <u>9208247</u>	80802121 <u>65887712</u>
80314927 <u>46349441</u>	8760051 <u>2273528</u>	4003347 <u>2712617</u>	82364354 <u>35845858</u>
8590323 <u>5962665</u>	42754101 <u>32851519</u>	2558496 <u>1412743</u>	93832917 <u>32105144</u>

Set B

These examples contain 90 subtraction facts, each occurring but once, if the Borrowing Method is used.

1	2	3	4	5	6
92780 <u>44146</u>	87314 <u>22349</u>	70031 <u>16598</u>	40033 <u>27127</u>	47823 <u>17358</u>	64354 <u>45858</u>
7	8	9	10	11	12
90320 <u>82667</u>	52785 <u>32859</u>	54121 <u>51549</u>	58496 <u>12743</u>	97442 <u>38465</u>	24814 <u>14136</u>
13	14	15	16	17	18
95977 <u>58859</u>	39866 <u>36939</u>	93697 <u>79436</u>	71125 <u>68237</u>	95126 <u>92717</u>	96162 <u>12967</u>

Multiplication

This test contains all the multiplication facts from 2×2 to 9×9 .

$\begin{array}{r} 4357 \\ 74 \\ \hline \end{array}$	$\begin{array}{r} 8692 \\ 37 \\ \hline \end{array}$	$\begin{array}{r} 7398 \\ 29 \\ \hline \end{array}$	$\begin{array}{r} 2645 \\ 92 \\ \hline \end{array}$
$\begin{array}{r} 2645 \\ 54 \\ \hline \end{array}$	$\begin{array}{r} 8739 \\ 45 \\ \hline \end{array}$	$\begin{array}{r} 7534 \\ 86 \\ \hline \end{array}$	$\begin{array}{r} 6982 \\ 68 \\ \hline \end{array}$

Division

This test requires 90 multiplication facts if the correct answers are obtained.

$71608 \overline{)60508760}$	$87160 \overline{)79838560}$	$39245 \overline{)17110820}$
$45239 \overline{)12576442}$	$53429 \overline{)50810979}$	$60871 \overline{)16617783}$

Common Fractions

Write all answers in lowest terms.

	a	b	c	d
1.	$\frac{3}{4} + \frac{1}{8} =$	$\frac{2}{3} + \frac{1}{6} =$	$\frac{1}{3} + \frac{1}{4} =$	$\frac{2}{3} + \frac{1}{9} =$
2.	$\frac{1}{2} - \frac{1}{4} =$	$\frac{3}{4} - \frac{5}{8} =$	$\frac{2}{5} - \frac{3}{10} =$	$\frac{3}{4} - \frac{2}{3} =$
3.	$\frac{1}{2} \times 3 =$	$\frac{2}{3} \times 4 =$	$\frac{3}{4} \times 4 =$	$\frac{5}{6} \times 12 =$
4.	$\frac{1}{2} \times \frac{2}{3} =$	$\frac{5}{8} \times \frac{8}{9} =$	$\frac{3}{5} \times \frac{2}{3} =$	$\frac{1}{4} \times \frac{1}{3} =$
5.	$4 \times 2\frac{1}{2} =$	$3\frac{1}{3} \times 3 =$	$2 \times 5\frac{3}{4} =$	$8 \times 2\frac{5}{12} =$
6.	$\frac{1}{2} \div \frac{1}{2} =$	$\frac{1}{2} \div \frac{1}{4} =$	$\frac{1}{8} \div \frac{1}{4} =$	$\frac{3}{4} \div \frac{1}{3} =$
7.	$\frac{1}{4} \div 8 =$	$\frac{3}{4} \div 6 =$	$\frac{5}{8} \div 3 =$	$\frac{1}{12} \div 6 =$
8.	$3\frac{3}{4} \div 3 =$	$12\frac{6}{7} \div 6 =$	$15\frac{5}{8} \div 5 =$	$24\frac{4}{5} \div 4 =$
9.	$18\frac{2}{3} \div 6 =$	$15\frac{1}{2} \div 4 =$	$12\frac{3}{4} \div 8 =$	$23\frac{1}{2} \div 12 =$
10.	$2\frac{1}{2} \times 2\frac{1}{2} =$	$3\frac{2}{3} \times 2\frac{1}{4} =$	$5\frac{1}{2} \times 2\frac{3}{4} =$	$12\frac{1}{8} \times 2\frac{1}{2} =$

Decimal Fractions

The only important skill that is peculiar to decimal fraction processes is ability to place with 100% accuracy the decimal point in the product or in the quotient.

Set A

Place the decimal point where it belongs in each of these products. Supply zero where it is necessary.

- | | |
|--------------------------------|----------------------------------|
| 1. $3.4 \times 6.5 = 2210$ | 11. $23.5 \times 2.5 = 5875$ |
| 2. $.68 \times .75 = 5100$ | 12. $34 \times .034 = 1156$ |
| 3. $4 \times .96 = 384$ | 13. $88.2 \times .81 = 71442$ |
| 4. $.04 \times 9.6 = 384$ | 14. $3.45 \times .09 = 3105$ |
| 5. $.025 \times .25 = 625$ | 15. $.03 \times 500 = 1500$ |
| 6. $.4 \times .88 = 352$ | 16. $3004 \times .004 = 12016$ |
| 7. $.07 \times .008 = 56$ | 17. $3.1416 \times 40 = 1256640$ |
| 8. $64 \times .64 = 4096$ | 18. $.26 \times 3.75 = 9750$ |
| 9. $12.5 \times 125 = 15625$ | 19. $.01 \times .001 = 1$ |
| 10. $13.5 \times 1.35 = 18225$ | 20. $100 \times .0001 = 100$ |

Set B

Place the decimal point in each of these quotients. Supply zero where it is necessary.

- | | | |
|--|---|--|
| 1. $\begin{array}{r} 275 \\ 27.5 \overline{)756.25} \end{array}$ | 4. $\begin{array}{r} 275 \\ 275 \overline{)756.25} \end{array}$ | 7. $\begin{array}{r} 275 \\ 2.75 \overline{)75.625} \end{array}$ |
| 2. $\begin{array}{r} 275 \\ .275 \overline{)7.5625} \end{array}$ | 5. $\begin{array}{r} 275 \\ 275 \overline{)7.5625} \end{array}$ | 8. $\begin{array}{r} 275 \\ 27.5 \overline{)7.5625} \end{array}$ |
| 3. $\begin{array}{r} 275 \\ 275 \overline{)7562.5} \end{array}$ | 6. $\begin{array}{r} 275 \\ 2.75 \overline{)75625} \end{array}$ | 9. $\begin{array}{r} 275 \\ .275 \overline{)756.25} \end{array}$ |

Set C

Find and check the answers to these examples in 6 minutes. Exchange your work with a classmate for verification.

a	b	c
1. $3.4 \times 6.5 = ?$	$.68 \times .75 = ?$	$4 \times .96 = ?$
2. $.04 \times 9.6 = ?$	$.025 \times .25 = ?$	$.4 \times .08 = ?$
3. $.07 \times .008 = ?$	$64 \times .64 = ?$	$12.5 \times 125 = ?$
4. $13.5 \times 1.35 = ?$	$.75 \times 75 = ?$	$.65 \times 65 = ?$
5. $6.5 \times .650 = ?$	$23.5 \times 2.5 = ?$	$34 \times .034 = ?$
6. $88.2 \times .81 = ?$	$3.45 \times .09 = ?$	$.03 \times 500 = ?$
7. $3004 \times .004 = ?$	$3.1416 \times 40 = ?$	$.26 \times 3.75 = ?$
8. $.01 \times .001 = ?$	$100 \times .0001 = ?$	$.002 \times 2000 = ?$

Cancellation Test

Find these results by cancellation. Remove large common factors. Do 10 right in less than 5 minutes.

a	b
1. $\frac{36 \times 27 \times 24}{9 \times 4 \times 6} = ?$	$\frac{25 \times 16 \times 7}{14 \times 12} = ?$
2. $\frac{18 \times 19 \times 17}{85 \times 57} = ?$	$\frac{17 \times 18 \times 19 \times 15}{85 \times 90} = ?$
3. $\frac{12 \times 15 \times 18}{72 \times 8} = ?$	$\frac{16 \times 21 \times 24}{14 \times 72} = ?$
4. $\frac{68 \times 72 \times 99}{24 \times 17 \times 11} = ?$	$\frac{18 \times 27 \times 36}{81 \times 54} = ?$
5. $\frac{13 \times 15 \times 12}{78 \times 60} = ?$	$\frac{24 \times 36 \times 45}{360} = ?$

I

Do 12 right in 8 minutes.

359	803	597	908	751	648	205
235	924	383	762	839	276	110
714	652	708	273	986	382	324
129	260	227	537	474	401	638
647	545	474	154	318	514	411
280	381	566	891	563	720	539
486	769	690	389	635	199	747
592	827	815	616	147	865	156
678	138	189	445	292	907	200

586	170	215	245	864	319	932
324	676	787	306	977	985	749
247	505	478	498	590	758	288
174	237	196	589	745	264	539
958	458	509	250	839	437	672
435	722	924	615	986	801	416
913	384	840	824	692	126	284
692	899	382	707	548	598	219
709	968	623	191	704	677	704

II

Add from left to right. Check from right to left.

1. $7 + 8 + 5 + 4 + 9 + 8 + 5 + 3 + 4 + 2 + 5 + 9 = \dots\dots\dots$
2. $11 + 8 + 6 + 4 + 7 + 2 + 3 + 4 + 7 + 8 + 15 + 20 = \dots\dots\dots$
3. $2 + 7 + 8 + 2 + 8 + 7 + 6 + 9 + 5 + 4 + 6 + 15 = \dots\dots\dots$
4. $8 + 4 + 5 + 7 + 4 + 5 + 4 + 15 + 16 + 8 + 7 + 3 = \dots\dots\dots$

III

The standard score for examples of this kind is 18 right in 10 minutes. Are you up to the standard?

a	b	c	d	e	f
1. 5226	9149	6268	9397	7337	8243
2883	8367	7725	6158	2674	6429
2584	251	8331	3732	9669	9298
58	7535	5493	4641	5114	7404
<u>2398</u>	<u>5223</u>	<u>3910</u>	<u>7919</u>	<u>8154</u>	<u>2575</u>
2. 7493	8937	8625	2123	5142	3691
9016	6345	4091	1679	376	4526
6487	2783	3844	5555	4955	7479
7591	4883	8697	6331	9314	2087
<u>6166</u>	<u>1341</u>	<u>7314</u>	<u>6808</u>	<u>5507</u>	<u>8165</u>
3. 8685	7521	2846	8253	7296	3580
4944	2754	6389	7594	6802	6295
5636	4856	3528	6031	3690	3782
3678	7039	3893	7426	3285	9061
<u>3869</u>	<u>9483</u>	<u>4678</u>	<u>7426</u>	<u>9735</u>	<u>3648</u>

IV

How long does it take you to add these examples and to check your answers?

\$559.39	\$313.44	\$144.45	\$123.44	\$619.29
393.63	145.15	454.23	312.34	314.36
<u>811.74</u>	<u>318.75</u>	<u>611.77</u>	<u>514.47</u>	<u>618.91</u>

A Common Fraction, Decimal, and Percentage Inventory Test

Time yourself. Try again tomorrow.

Study the question; then write the answer.

1. What is 4% of 75?
2. What is .04 of 80%?
3. What is 20% of 150?
4. What is .20 of 120?
5. What is $\frac{1}{5}$ of 140?
6. What is 25% of 160?
7. What is .25 of 140?
8. What is $\frac{1}{4}$ of 180?
9. What is $33\frac{1}{3}\%$ of 165?
10. What is $33\frac{1}{3}\%$ of 156?
11. What is $\frac{1}{3}$ of 195?
12. What is 40% of 90?
13. What is .4 of 90?
14. What is $\frac{2}{5}$ of 90?
15. What comm. frac. = 60%?
16. What $\% = \frac{3}{4}$?
17. What decimal = $37\frac{1}{2}\%$?
18. What comm. frac. = $37\frac{1}{2}\%$?
19. $\frac{5}{6}$ = what %?
20. 35% = what decimal?
21. 18 is 3% of what number?
22. 24 is 8% of what number?
23. 15 is 20% of what number?
24. 40 is 5% of what number?
What number is
25. 20% more than 60?
26. 25% more than 80?
27. 40% less than 150?
28. What is $\frac{3}{5}\%$ of \$100?
29. What is $\frac{3}{8}\%$ of \$1200?
30. How many mills in $\frac{2}{5}\%$ of \$1?
31. 120 is $\frac{4}{5}\%$ of what number?
32. $\frac{3}{5}\%$ of \$1 = how many cts.?
33. What is the no. if 6% of it = \$30?
34. What is the no. if 5% of it = \$60?
35. What is the no. if .12 of it = \$150?
36. 75 is 5% of what number?
37. $54 = .06 \times$ what number?
38. $120 = 6\%$ of what number?
39. .12 of what number = 180?
40. $175 = .25 \times$ what number?
41. $\$15 = 1\frac{1}{2}\%$ of what number?
42. $\$36 = \frac{3}{5}\%$ of what number?
43. What is $2\frac{1}{2}\%$ of \$450?
44. What is $3\frac{1}{3}\%$ of \$780?
45. 80 is .16 of what number?
46. What is \$80, less 10% and 10 off?
47. Find $.00\frac{1}{2}\%$ of \$10000.
48. $\frac{3}{4} - \frac{3}{4}\% = ?$
49. $\frac{3}{4}$ of \$10 - 7.5% of \$100 = ?
50. 10% of 10% of 10% = ?

Percentage

Write the number of the example and its answer.

1. 6% of 50 = ...
2. 50 is % of 200.
3. 8 is % of 12.
4. 8% of \$23 = ...
5. $.20 \times \$20 = \dots$
6. $.32 \times \$200 = \dots$
7. 20 is 25% of
8. 10% of 30 = ... % of 8.
9. 15% of 12 ft. = ...
10. $45^\circ = \dots \% \text{ of } 90^\circ$.
11. $12\frac{1}{2}\%$ of a bu. = ... qt.
12. $37\frac{1}{2}\%$ of 24 hr. = ...
13. \$6 is 12% of
14. 3 yd. is 25% of
15. $\frac{1}{2}$ is % of $\frac{1}{4}$.
16. $\frac{1}{2}$ is % of .5.
17. A rt. $\angle = \dots \% \text{ of a straight } \angle$.
18. Each angle of an equilateral $\triangle = \dots \% \text{ of } 180^\circ$.

The Test of Life

This test is given to those seeking employment in a certain department store in a large city. The passing mark is 100. The time must not exceed 10 minutes.

Try to solve all right in less than 10 minutes.

- | | | |
|---|--|---|
| <p>1. 98773
 37521
 46378
 73216
 53764
+ 21976

 Ans.</p> | <p>3. 75412
 $\times 8$

 Ans.</p> | <p>6. Find the cost of each of the following:
(1) 9 yd. gingham @ $12\frac{1}{2}\text{¢}$ Ans.</p> |
| <p>2. 74621
- 43130

 Ans.</p> | <p>4. 98744
 $\times 78$

 Ans.</p> | <p>(2) $12\frac{1}{2}$ yd. muslin @ 22¢ Ans.

(3) 4 pr. lace curtains @ \$3.75 pr.
Less 10% discount..... Ans.</p> |
| <p>5. 9)13437659</p> | | |
| <p>(4) $15\frac{1}{4}$ yd. lace @ $12\frac{1}{2}\text{¢}$ Less 25% discount Ans.</p> | | |

Reviewing Surfaces and Solids

Write your answer in good English.

Remember that some of these questions may have more than one answer.

Construct the figure if it will help you to answer the question.

What can I find and how can I find it if I know:

1. The length of a cube?
2. The radius of a circle?
3. The side of a square?
4. The base of a right isosceles triangle?
5. The circumference of a circle?
6. The area of one face of a cube?
7. The diameter of the base and the altitude of a cylinder?
8. The base and altitude of a right triangle?
9. The base and altitude of a rectangle?
10. The radius of a sector and its central angle?
11. The dimensions of a rectangular solid?
12. The radius and length of a cylinder?
13. Two angles of a triangle?
14. One of the acute angles of a right triangle to be 45° ?
15. The bases and altitude of a trapezoid?
16. The diagonal of a trapezium and the two altitudes drawn to it?
17. The dimensions of a rectangular coal bin?
18. The radius of a sector and its arc in feet or inches?
19. The diameter of a circle?
20. A circle and its circumscribed square?

1. How many degrees are there in the angle made by the minute hand and hour hand at 3 o'clock? at 6 o'clock? at 1 o'clock? at 4 o'clock?

HINT.—If necessary use a clock face or watch face to help you.

2. Through how many right angles does the minute hand of a clock move from 4 o'clock to 4:45 o'clock?

3. Through how many right angles does the hour hand of a clock move from 3 o'clock to 9 o'clock? from 10 o'clock to 7 o'clock?

4. Through how many degrees does the minute hand move in 60 min.? in 10 min.? in 25 min.? in 5 min.?

5. How many degrees in the angle formed by the hands of a clock at 11 o'clock? at 5 o'clock? at 7 o'clock? at 9 o'clock? at 10 o'clock?

6. Each of the questions in No. 5 has two answers. Do you see why?

7. What per cent of a right angle is an angle of 60° ?

8. What per cent of a 45° angle is a right angle?

9. How many degrees are there on the horizon between due west and due south? Between southeast and due north?

10. How many degrees are there on the horizon between due south and due northwest? Between due north and due southeast? Between southeast and southwest?

11. How many degrees are there between any point on the horizon and the point directly over head (Zenith)?

12. How many degrees above the horizon is the north star in your locality?

A True and False Test

On a sheet write the numbers in order from 1 to 22. Read carefully each of these equations. Place a check mark on your sheet after those numbers which enumerate true equations. Change those which you think are false to true ones. Write only the correct equation after its proper number.

1. $8 \div \frac{1}{2} = \frac{1}{2}$ of 8.

2. $\frac{2}{3} \times 6 = \frac{2}{3}$ of 6.

3. $1000 \times .016 = 16$.

4. $.05 + .06 = .011$.

5. $\frac{3}{4} \div \frac{2}{3} = \frac{4}{3} \times \frac{2}{3}$.

6. $10 \div .04 = 250$.

7. $\$ \frac{5}{8} = 625\text{¢}$.

8. 1 bu. = 32 qt.

9. 1 mi. = 1660 rd.

19. $6 \div \frac{1}{3} = 2$

20. In any rt. $\triangle h^2 = a^2 + b^2$.

21. In any $\triangle \angle A + \angle B + \angle C = 2\text{rt. } \angle s$.

22. $\sqrt{36+49} = \sqrt{36} + \sqrt{49}$.

10. Area $\odot = \pi D^2$.

11. Area $\triangle = \frac{1}{2} ab$.

12. Area $\triangle = a \times \frac{1}{2} b$.

13. Area $_{sector} = \text{arc} \times \frac{1}{2} R^2$.

14. $6\frac{1}{2} \div 2\frac{1}{2} = 2\frac{1}{2}$.

15. $\frac{1}{4} \times \$18 = \$18 \div 4$.

16. $18\text{¢} \div 3\text{¢} = \frac{1}{3}$ of 18¢.

17. $I = P \times R \times T$.

18. Loss = sell.pr. - cost.

Same-different Test

On a narrow slip of paper provide for two columns. Make the heading of one column "Same"; of the other "Different."

Read each pair of numbers on the next page carefully. Then write the number which designates each pair in the proper column on your slip. No. 1 will go under "Same" because $\frac{2}{3} = .66\frac{2}{3}$, but number 2 will go under "Different" because $\$ \frac{1}{10}$ does not equal .1 dime.

1. $\frac{2}{3}$	$.66\frac{2}{3}$	11. $12\frac{1}{2}\text{¢}$	\$.125
2. \$ $\frac{1}{10}$	.1 dime.	12. $2R$	D
3. $5\frac{1}{2}$ yd.	1 rd.	13. $\frac{\pi D^2}{4}$	πR^2
4. 160 rd.	1 mi.	14. $D \div \pi$	R
5. $16\frac{2}{3}\text{¢}$	$16\frac{2}{3}\%$	15. $C \times R$	πR
6. $\frac{1}{2}$	$\frac{1}{2}\%$	16. πD	C
7. $.00\frac{1}{2}$	$\frac{1}{2}\%$	17. $2\frac{1}{2}\%$	$\frac{1}{40}$
8. .3	.30	18. $3\frac{1}{3}\%$	$.03\frac{1}{3}$
9. 017.	.017	19. 2.5	$\frac{1}{4}$
10. 012.7 mi.	12.7 mi.	20. .25	.250

Triangle Completion Test

Read carefully, then supply the missing number, word, or words.

1. The sum of the 3 angles of a triangle is.....°.
2. The size of each angle of an equilateral triangle is.....°.
3. angles of an isosceles triangle are equal.
4. The largest angle of a right triangle is.....°.
5. The sum of the two acute angles of a right triangle is.....°.
6. Each angle of an acute triangle is less than.....°.
7. A scalene triangle has.....sides equal.
8. An isosceles triangle has.....sides equal.
9. A triangle with three sides equal is called.....
10. The hypotenuse of a right triangle whose other sides are 5 and 12 is.....

FORMULA TESTS**Writing Formulas**

Write the formula which tells

1. The relation of the hypotenuse of a right triangle to the other sides.
2. The area of a triangle.
3. The area of a rhomboid.
4. The convex surface of a cylinder.
5. The area of a square in terms of its diagonal.
6. The entire surface of a cube.
7. The volume of a prism.
8. The area of a circle in terms of its diameter.
9. The area of a trapezoid.
10. The area of a sector in terms of its arc and radius.
11. The angle sum of any triangle.

Testing Formulas

1. Is this formula true, $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$? Why?
2. Check your answer for number one by using these values: $a = .5$, $b = .25$, and $c = .75$.
3. Is this formula true, $\frac{b}{a} + \frac{b}{c} = \frac{2b}{a+c}$? Why?
4. Check your answer for number 3 by using these values: $a = .12$, $b = .3$, and $c = .18$.
5. Is this formula true, $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$? Why?

6. Check your answer for number 5 with these values:
 $a=6$, $b=4$ $c=3$.

7. Is this formula true, $\frac{b}{a} - \frac{b}{c} = \frac{0}{a-c}$? Why?

8. Check your answer for number 7 with the values in number 6.

9. Is this formula true, $\frac{a}{c} \times \frac{b}{d} = \frac{ab}{cd}$? Why?

10. Check your answer for number 9 with these values:
 $a=.2$, $b=.3$, $c=.02$, and $d=.03$.

Seeing Relations in Formulas

1. In the formula, $I=P \times R \times T$, what will happen to I if P is doubled? What will happen to I if both P and R are doubled? What if P , R , and T are doubled?

2. Check your answers in number one for these values:
 $P=\$200$, $R=3\%$, and $T=2$ yr.

3. In the parcel post formula for zone 8, c (cost in cents) $=12w$ (w =wt. in lb.), what happens to c if w is multiplied by 4? Prove your answer.

4. In the parcel post formula for zone 5, $c=6w+2$, is the cost doubled when the weight is doubled? Why?

5. In the formula, $A \square = l \times w$, what will happen to A if l is doubled? What if l and w are doubled?

6. Test your answers to number 5 by giving number values to l and w .

7. If each side of a square is doubled what happens to the area? What happens to the perimeter?

8. In the formula, $A \odot = \pi R^2$, how will it affect the value of A if R is multiplied by 3?

9. What happens to the volume of a cube if its dimensions are doubled?

10. Compare a 4-inch cube with a 2-inch cube.

(1) as to volume. (2) as to entire surface.

11. What fraction of an 8-inch cube remains after a 2-inch cube is cut out of one corner?

Mathematical Knowledge Test

Write a correct statement, formula, or equation, or make a drawing, so as to explain each of the following: (1) straight angle, (2) trapezoid, (3) isosceles triangle, (4) scalene triangle, (5) πR^2 , (6) lwh , (7) perpendicular, (8) isosceles right triangle, (9) $P \times R \times T$, (10) currency, (11) standard time, (12) gross profit, (13) overhead, (14) parallel lines,

(15) acute triangle, (16) obtuse triangle, (17) $\frac{a}{2} \times b$,

(18) $a \times b$, (19) $a \times \frac{(b+b')}{2}$, (20) $d \times \frac{d'}{2}$, (21) perimeter,

(22) rhombus, (23) equilateral triangle, (24) compound interest, (25) brokerage, (26) premium (when speaking of insurance), (27) promissory note, (28) check, (29) coupon, (30) market value, (31) face value, (32) proceeds of a note, (33) blank indorsement, (34) board foot, (35) a ratio, (36) interest, (37) π , (38) acute angle, (39) trapezium,

(40) $\frac{C \times R}{2}$.

1. If you sold a customer several bills of goods and the customer made several payments, how could you determine the amount still due?

2. Henry Williams made six different deposits in a bank at various times and issued ten different checks drawing money out of the bank. How could you find the balance in the bank, and what convenient arrangement would show him the facts?

3. If you know the balance in the bank at the end of May, the amount deposited, and the amount withdrawn during the month of May, how can you find the balance in the bank at the beginning of May?

4. If you know the total cost of six articles and the individual cost of five of them, how would you find the cost of the remaining one?

5. Williams Brothers bought a certain amount of goods during the year 1923. At the end of the year you know the cost of the goods unsold. How will you find the cost of the goods that they sold?

6. During the year 1923, Hannan & Co. made a profit from selling goods and a profit from renting part of their store. They were at an expense in hiring clerks, advertising their goods, paying for fuel, light, etc. If you knew the amounts in each of the above items, how would you find the net profits?

7. If you knew the distance from St. Louis to New Orleans and knew how far a gallon of gasoline would run an automobile, and knew the cost per gallon, how would you estimate the cost of gasoline from St. Louis to New Orleans?

Reviewing Graphs

1. Figures 1 to 5 on the next page represent line, bar, circle, pictorial, and rectangle graphs. Give to each figure the correct name.

2. After studying these graphs tell what each one says.

3. In figure 5 compare the lira with the franc; with the U. S. dollar.

4. In figure 3 what temperature does *N* represent? Are the degrees shown in this graph Fahrenheit or Centigrade?

5. In figure 1 what per cent of baby robin food is "other insects"?

6. In 1920 the world's corn crop was 4,144,000,000 bu. What was the crop of the United States for that year?

7. In figure 4 what was the rainfall on the 10th day? How much on the 11th day? What is the ratio of the rainfall on the 13th day to that of the 9th day? What is the ratio of the rainfall on the 12th day to that of the 13th day?

8. Sugar was sold per lb. at these prices: 1914, 5¢; 1915, 6¢; 1916, $6\frac{1}{2}$ ¢; 1917, 7¢; 1918, 9¢; 1919, 12¢; 1920, 22¢. Show with a graph these changes in price from year to year.

9. Make a graph showing the prices for Thanksgiving turkey, which sold per lb. as follows: 1916, 30¢; 1917, 32.5¢; 1918, 35¢; 1919, 44¢; 1920, 56¢.

10. Construct the graph showing the attendance record of your own class for two weeks.

11. Show with a graph what happens to the area of a circle when the diameter increases.

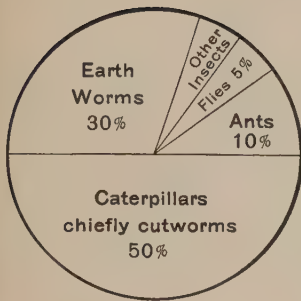


Fig. 1

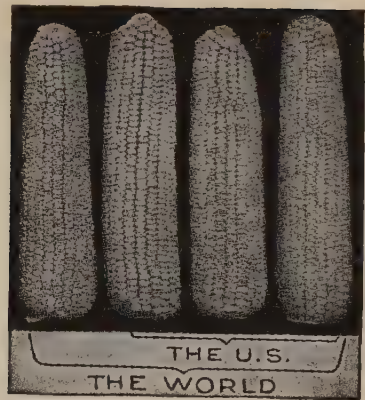


Fig. 2

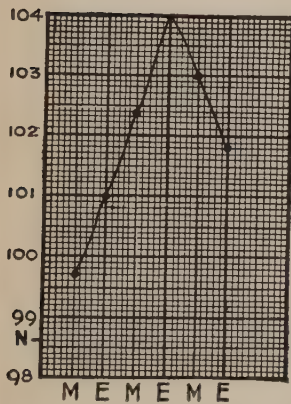


Fig. 3

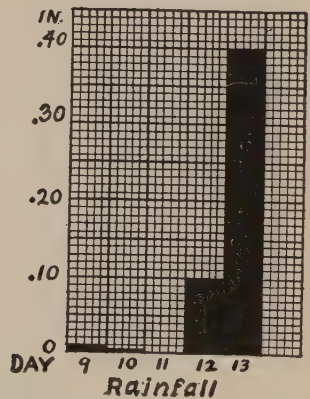


Fig. 4

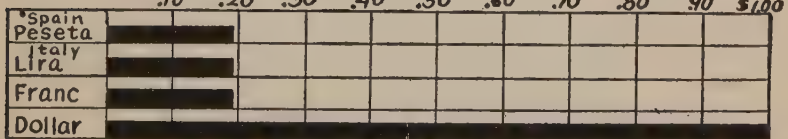
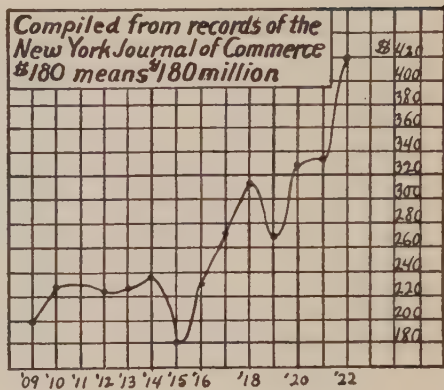


Fig. 5

I. It has been said that as a nation we have not learned to build wisely, to inspect our premises frequently, and to keep our homes, stores, and factories free from rubbish. As a consequence two out of every three fires are the result of carelessness, that is, are preventable.



The highest fire loss that ever occurred in the United States for any one year amounted to \$460,000,000 in 1906, for which the San Francisco disaster was largely responsible. There is no such disaster to account for the great loss in 1922.

Read the above graph carefully; then solve these problems.

1. Compare the average daily fire loss in 1922 with that in 1915.
2. The yearly fire loss in Great Britain is 72¢ per capita. Compare this with our per capita loss in 1920. Call the 1920 population in the United States 105,000,000.
3. Compare the total loss of the first six years in the graph with that of the last six years.
4. What was the total destruction of wealth by fire from 1909 to 1922?

5. What per cent is the total fire loss from 1909 to 1922 of the total deposits, \$6,018,000,000, in all the savings banks in the United States in 1921?

6. What may young people do to reduce fire losses?

II. A dollar will carry a ton of freight over a wagon road 4 mi.; over railroad at the average rate for United States railways, 132 mi.; on the Erie Canal, 332 mi.; on European canals, 500 mi.; at the average rate through Soo Canal in 1911, 1500 mi.; at the coal rate on the Great Lakes and on the Ohio and Mississippi Rivers, 3000 mi.

1. Using the scale one millimeter equals 4 mi., draw horizontal bar graphs to show the relative freight charges in terms of mileage of the above types of transportation.

2. Find out how far one ton of freight can be carried for one cent over each of the above types of transportation.

3. What change must you make in your scale in No. 1 so as to allow the graphs which you drew to represent the mileage found in No. 2?

III. The fastest airplane speed to September, 1923, was made at Mt. Clemens, Mich., Oct. 24, 1922, the rate being 248.5 mi. per hour. The fastest automobile speed was made at Daytona Beach, Fla., Apr. 10, 1922, the rate being 180.27 mi. per hour. The fastest motorcycle speed was 133.3 mi. per hour, being made by a Glen Curtis motorcycle in 1911.

1. Draw bar graphs showing these speeds.

2. Draw a graph showing the above speeds on the basis of the number of seconds required to travel one mile.

A Practice Test in Percentage

Seven right in 30 minutes is a good record.

1. A merchant buys goods listed at \$962.50. One firm offers discounts of 20, 10, and 5%. Another firm offers 5, 10, and 20% off. Which offer shall he accept? Explain.

2. An article is listed at \$150 by two different firms. One offers discounts of $33\frac{1}{3}$, 20, and 10%, and the other offers discounts of 40, 10, and 10%. Which offer is the better? How much?

3. If a bushel of oats sells for $53\frac{1}{2}\text{¢}$, which is better for the broker, a commission of 2% or a commission of $1\frac{1}{4}\text{¢}$ per bu.? How much better on 864 bu.?

4. A creditor receives from a bankrupt \$1,600, which is 40% of his claim. What is the amount of his claim?

5. This year my sales were \$12,000, which was an increase of 20% over last year. What were my sales last year?

6. A lot was purchased in March, 1922, for \$3,500 and was sold one year later for \$3,850. What was the per cent of profit on the cost price? What was the lot worth in March, 1924, if its value increased at the same rate?

7. A certain department store advertised that the management would refund the amount of car fare one way to all customers within 100 miles who would come to the store and buy \$25 worth of goods. One woman living 75 miles from this store bought \$32.58 worth of goods. The car fare refunded was at the rate of 3.6¢ per mile. To what rate of discount was this equivalent?

Solve the problems based on this lumber bill.

Lot	No. of Pieces	Size in Inches	Length in Feet						Total
			12	14	16	18	20	24	
No. 1	10	12×1	?	?	?	?	?	?	?
No. 2	12	$12 \times \frac{3}{4}$	?	?	?				?
No. 3	6	12×2	?	?	?	?			?
No. 4	4	8×2	?	?		?		?	?
No. 5	9	4×3	?	?	?	?		?	?
No. 6	6	4×4	?	?	?			?	?
No. 7	8	6×3	?	?	?	?	?	?	?
No. 8	8	8×6	?	?	?	?			?
No. 9	9	8×8	?	?		?		?	?
No. 10	2	6×6	?	?	?	?	?	?	?
No. 11	1	12×12			?	?	?	?	?
			?	?	?	?	?	?	??

1. On a separate sheet make a chart as indicated above.
2. After the indicated chart is made, find the answers to the nearest board foot for the question marks, making computations with pencil only when necessary.
3. How many board feet in each lot?
4. Find the cost of all the 12 ft. lumber at \$40 per M.
5. How many board feet of 14 ft. lumber?
6. What is the cost of the 24 ft. lumber at \$60 per M?
7. Find, in two ways, the total number of board feet required by all the question marks.

The Reavis Problem Test without Figuring

An 8th grade pupil good in problem solving should write 9 correct answers in 12 minutes (1924 standard). After the test solve those you did not try; also those you missed.

1. A submarine made a voyage of a given number of miles, going a certain number of miles under water and the remainder of the distance on the surface. How would you find the distance traveled on the surface?

2. A grocer received a bill giving the number of pounds shipped and the cost of an order of sugar. If you were given this bill, how would you find the cost of the sugar per pound?

3. If you know the weight of dough required to make a single loaf of bread, how would you find the weight of the dough a baker must prepare to make a given number of similar loaves?

4. A fruit dealer bought a stalk of bananas containing a certain number of dozens for a certain price. How would you find the cost per dozen?

5. If you know the cost of my purchase at a store and the amount I had when I left the store, how would you find the amount of money I had when I went to the store?

6. If you know the number of brick carried by a hod carrier at a load and the weight of the load in pounds, how would you find the weight of a single brick?

7. A man bought a house and lot for a certain price. He paid a certain amount in cash and gave a mortgage for the rest. How would you find the amount of the mortgage?

8. If you know the number of hours required by a train to go from one city to another and the speed of the train per hour, how would you find the distance between the cities?

9. At a school picnic the patrons gave a treat of ice cream to each child present. If you knew the number of children a gallon of ice cream would serve and the number of children present, how would you find the number of gallons required to treat all the children?

This is the Rice test given to 689 8th grade children. The average achievement (right answers) for the best class was 91.7%; for the poorest class it was 11.3%; the average for all the classes was 49.4%.

Try to solve 7 right in 50 minutes.

Find out if your class stands above or below the average.

1. The salt water which was obtained from the bottom of a mine of rock salt contained 0.08 of its weight of pure salt. What weight of salt water was it necessary to evaporate in order to obtain 3,896 pounds of salt?

2. If a map 10 inches wide and 16 inches long is made on a scale of 50 miles to the inch, what is the area in square miles that the map represents?

3. A gentleman gave away $\frac{1}{7}$ of the books in his library, lent $\frac{1}{6}$ of the remainder, and sold $\frac{1}{5}$ of what was left. He then had 420 books remaining. How many had he at first?

4. Gunpowder is composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts. How much of each in 360 pounds of powder?

5. The insurance on $\frac{2}{3}$ of the value of a hotel and furniture cost \$420. The rate being 70 cents on \$100, what was the value of the property?

6. Steel was sold at \$27.60 a ton, with a profit of 15%, and a total profit of \$184.50. How much was sold?

7. A man sold 50 horses at \$126 each. On one-half of them he made 20 per cent of the cost, and on the other half he lost 10 per cent of the cost. How much did he gain?

8. A fruit dealer bought 300 apples at the rate of 5 for a cent, and 300 at 4 for a cent. He sold them all at the

rate of 8 for 5 cents. What per cent did he gain on his investment?

Problems about American Education

1. The general population of the United States increased 68% from 1890 to 1920. For every 100 persons in 1890 there were how many in 1920?

2. The increase in enrollment in elementary and high schools in the United States from 1890 to 1920 was 139%. For every 100 elementary and high school pupils in 1890 there were how many in 1920?

3. The growth (increase) in high school enrollment for the above period was 984%. For every 100 high school pupils in 1890 there were how many in 1920?

4. In the colleges and universities the increase in enrollment from 1890 to 1920 was 432%. For every 100 college and university students in 1890 there were how many in 1920?

5. If the total high school enrollment in 1890 was 202,983, find the total in 1920. See problem 3.

6. The total enrollment in American colleges and universities in 1890 was 67,094. Find the total enrollment in 1920. See problem 4.

7. Compare the rate of growth of high school enrollment with that of the general population for the above period.

8. Compare the rate of growth of college and university enrollment with that of the general population for the above period.

An 8th Grade Completion Test

On another sheet write the number of the statement or example and the answer.

1. $34 + \text{---} = 60$; $12 + 18 + \text{---} = 40$.
2. $18 - \text{---} = 12$; $\text{---} - 15 = 36$.
3. $2\frac{1}{2} \times 6 = \text{---}$; $10 \div 2\frac{1}{2} = \text{---}$.
4. $\frac{2}{3} + \frac{3}{4} = \text{---}$; $\frac{3}{4} - \frac{2}{3} = \text{---}$.
5. $\frac{2}{3} \times \frac{3}{4} = \text{---}$; $\frac{2}{3} \div \frac{3}{4} = \text{---}$.
6. 1% of \$200 = --- ; 3% of 80 ft. = --- .
7. $4 \div .4 = \text{---}$; $.8 \div .2 = \text{---}$.
8. Draw a line which you think is 1 in. long.
9. Draw a rectangle 2 in. long and 1 in. wide.
10. Draw a right triangle.
11. There are --- things in a dozen; --- lb. in a ton.
12. There are --- days in a common year; --- days in an interest year.
13. It takes --- ft. to make a rod; --- in. to make a yard.
14. There are --- sq. in. in a sq. ft.; --- sq. rd. in an acre.
15. There are --- cu. in. in a gallon; --- cu. in. in a bushel.
16. April has --- days; December has --- days.
17. The interest on \$200 for 3 years at 5% is \$ --- .
18. The amount on \$400 for 4 years at 4% is \$ --- .
19. A room 30 ft. $\times$ 20 ft. $\times$ 12 ft. contains --- cubic feet of air.

20. The ceiling of this room contains about ____ square feet.

21. This room contains about ____ cubic feet of air.

22. At 3 for a cent 12 marbles will cost ____.

23. At 2 for 5¢ you can buy ____ pencils for 20¢.

24. 2% of \$1,000 is the same as 4% of \$____.

25. If a box 6 ft. long and 4 ft. wide contains 72 cu. ft., its depth is ____ ft.

26. If A has \$16, B $\frac{3}{4}$ as much as A, and C $\frac{1}{2}$ as much as B, they all have \$____.

27. An ocean steamer using 300 tons of coal in 24 hr. uses ____ tons in 8 hr.

28. If $\frac{9}{10}$ of my money is \$72, then I have \$____.

29. If I earn \$10 a week collecting bills at 2% commission, then I must collect \$____ per week.

30. If a commission man sells \$2,000 worth of cotton for me at 2% commission, he should send me \$____.

31. If I wish a commission merchant to buy \$3,000 worth of sugar for me at 2% commission, then I should send him \$____.

32. A man sold an automobile which cost him \$1,600 at a loss of 20% of the cost. He received \$____ for his automobile.

33. A man sold his farm for \$12,000 at a gain of 10% of the selling price. The farm cost him \$____.

34. A pile of wood 16 ft. long, 4 ft. wide and 4 ft. high contains ____ cords.

35. Multiplying the numerator and denominator of a fraction by the same number ____ ____ ____ its value.

36. Multiplying the numerator and denominator of a fraction by the same number makes the number of parts _____ and the size of each part _____.

37. Dividing the denominator of a fraction by 2 makes the size of each part _____.

38. Dividing the numerator of a fraction by 3 makes the number of parts _____.

39. Multiplying the numerator of a fraction by 3 and dividing the denominator by 3 makes the number of parts _____ and the size of each part _____.

40. Dividing the numerator of a fraction by 2 and multiplying the denominator by 2 makes the number of parts _____ and the size of each part _____.

41. The exact interest on \$365 at 6% from Jan. 1 to April 11, 1922, was _____.

42. My house is assessed at \$8,000; my tax at \$2.35 a hundred is \$_____.

43. Insurance on 80% of the value of a house worth \$20,000 at $\frac{1}{2}\%$ costs \$_____.

44. If I buy 10 shares of stock paying 6% dividend yearly at 75, they cost me \$_____.

45. My annual income on the stock is \$_____.

46. The rate of income on the investment is _____%.

47. If I buy a 5% one thousand dollar bond due in 10 years at 92, the bond costs me \$_____.

48. I collect \$_____ of bond interest every year.

49. My annual rate of interest on the investment is _____%.

50. If I buy a 12% stock at 150, my rate of income on the investment will be ____ %.

51. What may I pay for an 8% stock to yield 6%?

52. What may I pay for a 6% stock to yield 8%?

53. The area of a trapezoid whose bases are 6 in. and 8 in. and whose altitude is 10 in. is ____.

54. The area of a rhombus whose diagonals are 8 in. and 10 in. is ____.

55. The convex surface of a cylinder whose altitude is 7 in. and the radius of the base 5 in. is ____.

56. The volume of the above cylinder is ____.

57. The convex surface of a cone whose radius is 3 in. and whose slant height is 5 in., is ____.

58. The surface of a sphere whose radius is 10 in. is ____.

59. The volume of the same sphere is ____.

60. (1) $1^{\circ}\text{F.} = \text{____ C.}$ (2) $1^{\circ}\text{C.} = \text{____ F.}$

61. (1) $40^{\circ}\text{F. temperature is ____ C.}$ (2) $50^{\circ}\text{C. temperature is ____ F.}$

62. A meter is ____ in.; a kilometer is about ____ of a mile.

63. A kilogram is ____ lb.; a liter is ____ qt.

64. A \$600, non-interest bearing, 90-day note discounted at 8% on the day of date of note brings \$ ____.

65. The net proceeds of an invoice of goods whose list price is \$600 with 20% and 10% off are \$ ____.

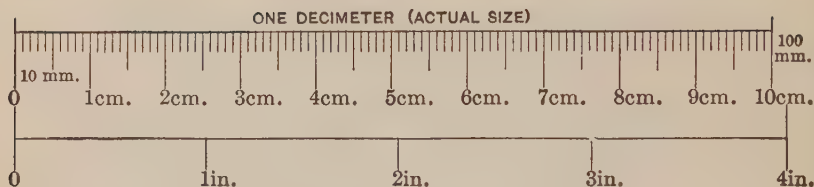
66. A gain of 25% on the cost is ____% on the selling price.

A Table of Imperfect Squares, showing the Square Roots to Two Decimal Places

Square	Square Root	Square	Square Root	Square	Square Root	Square	Square Root
2	1.41	40	6.32	75	8.66	110	10.48
3	1.73	41	6.40	76	8.71	111	10.53
5	2.23	42	6.48	77	8.77	112	10.58
6	2.44	43	6.55	78	8.83	113	10.63
7	2.64	44	6.63	79	8.88	114	10.67
8	2.82	45	6.70	80	8.94	115	10.72
10	3.16	46	6.78	82	9.05	116	10.77
11	3.31	47	6.85	83	9.11	117	10.81
12	3.46	48	6.92	84	9.16	118	10.86
13	3.60	50	7.07	85	9.22	119	10.90
14	3.74	51	7.14	86	9.27	120	10.91
15	3.87	52	7.21	87	9.32	122	11.04
17	4.12	53	7.28	88	9.38	123	11.09
18	4.24	54	7.34	89	9.43	124	11.13
19	4.35	55	7.41	90	9.48	125	11.18
20	4.47	56	7.48	91	9.53	126	11.22
21	4.58	57	7.55	92	9.59	127	11.26
22	4.69	58	7.61	93	9.64	128	11.31
23	4.79	59	7.68	94	9.69	129	11.35
24	4.89	60	7.74	95	9.74	130	11.40
26	5.09	61	7.81	96	9.79	131	11.44
27	5.19	62	7.87	97	9.84	132	11.48
28	5.29	63	7.93	98	9.89	133	11.53
29	5.38	65	8.06	99	9.95	134	11.57
30	5.47	66	8.12	101	10.04	135	11.61
31	5.56	67	8.18	102	10.09	136	11.66
32	5.65	68	8.24	103	10.14	137	11.70
33	5.74	69	8.30	104	10.19	138	11.74
34	5.83	70	8.36	105	10.24	139	11.79
35	5.91	71	8.42	106	10.29	140	11.83
37	6.08	72	8.48	107	10.34	141	11.87
38	6.16	73	8.54	108	10.39	142	11.91
39	6.24	74	8.60	109	10.44	143	11.95

NOTE.—For the square root of any number 100 times any number in the table, multiply the root by 10. Since the square root of 27 is 5.19, the square root of 2700 is 51.9. For the approximate square root of any number 10 times any number in the table, multiply the root by $3\frac{1}{2}$. The square root of 33 is 5.74. The square root of 330 is about $3\frac{1}{2} \times 5.74$ or nearly 18.17.

TABLES FOR REFERENCE



Measures of Length

English	Metric
12 inches (in.) = 1 foot (ft.)	10 millimeters (mm.) = 1 centimeter (cm.)
3 feet = 1 yard (yd.)	10 centimeters = 1 decimeter (dm.)
5½ yards, or 16½ ft. = 1 rod (rd.)	10 decimeters = 1 meter (m.)
320 rods = 1 mile (mi.)	10 meters = 1 dekameter (Dm.)
5280 feet = 1 mile	10 dekameters = 1 hektometer (Hm.)
1760 yards = 1 mile	10 hektometers = 1 kilometer (Km.)
63360 inches = 1 mile	1 meter = 40 in. (more accurately 39.37 in.)

Measures of Weight

English (Avoirdupois)	Metric
437½ grains (gr.) = 1 ounce (oz.)	10 milligrams (mg.) = 1 centigram (cg.)
16 ounces (oz.) = 1 pound (lb.)	10 centigrams = 1 decigram (dg.)
100 pounds (lb.) = 1 hundredwt (cwt.)	10 decigrams = 1 gram (g.)
2000 pounds = 1 ton (T.)	10 grams = 1 dekagram (Dg.)
2240 pounds = 1 long ton	10 dekagrams = 1 hektogram (Hg.)
196 pounds = 1 barrel (bbl.) of flour	10 hektograms = 1 kilogram (Kg.)
7000 grains = 1 pound	1 kilogram = 2.2 pounds.

Measures of Capacity or Volume

English	Metric
Liquid Measure	10 milliliters (ml.) = 1 centiliter (cl.)
2 pints (pt.) = 1 quart (qt.)	10 centiliters = 1 deciliter (dl.)
4 quarts (qt.) = 1 gallon (gal.)	10 deciliters = 1 liter (l.)
231 cu. in. = 1 gallon (gal.)	10 liters = 1 dekaliter (Dl.)
Dry Measure	10 dekaliters = 1 hektoliter (Hl.)
2 pints (pt.) = 1 quart (qt.)	10 hektoliters = 1 kiloliter (Kl.)
8 quarts (qt.) = 1 peck (pk.)	1 liter = about { 1.1 liquid qt. .9 dry qt. 64 cu. in.
4 pecks (pk.) = 1 bushel (bu.)	1 hektoliter = about 2.8 bu.
2150.42 cu.in. = 1 bushel (bu.)	

Measures of Surface

144 square inches (sq. in.)	160 square rods
= 1 square foot (sq. ft.)	= 1 acre (A.)
9 square feet	640 acres = 1 square mile (sq.mi.)
= 1 square yard (sq. yd.)	36 sq. mi. = a township (tp.)
30½ sq. yd. or 272½ sq. ft.	
= 1 square rod (sq. rd.)	

Measures of Volume or Capacity

1728 cubic inches (cu. in.)	38 cubic feet (approximately)
= 1 cubic foot (cu. ft.)	= 1 ton soft coal
27 cubic feet	34½ cubic feet (approximately)
= 1 cubic yard (cu. yd.)	= 1 ton hard coal
128 cubic feet	1 cu. ft. = .8 bushel
= 1 cord of 4 ft. wood	1 cu. ft. = .63 heaped bushel
	1 cu. ft. = .43 bu. of corn in ear

Measures of Time

60 seconds (sec.)	= 1 minute (min.)
60 minutes (min.)	= 1 hour (hr.)
24 hours (hr.)	= 1 day (da.)
7 days (da.)	= 1 week (wk.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year
360 days	= 1 interest year
100 years	= 1 century
10 years	= 1 decade

Measures of Angles or Arcs

60 seconds (")	= 1 minute (')
60 minutes (')	= 1 degree (°)
90 degrees (°)	= 1 quadrant
360 degrees (°)	= 1 circumference
π (pi)	= 3.1416 or 3¼
Century years	divisible by 400
and other years	divisible by 4 are leap years

Table of Equivalents Used in Recipes

3 teaspoonfuls (tsp.)	= 1 tablespoonful (tbsp.)
4 tablespoonfuls (tbsp.)	= ¼ cup or ½ gill.
2 gills	= 1 cup
2 cups	= 1 lb.
4 cups flour	= 1 lb.
3 cups meal	= 1 lb.
2 tbsp. sugar	= 1 oz.
2 cups granulated sugar	= 1 lb.)
2 cups solid butter	= 1 lb.

Values Accurate Enough for Ordinary Computation

1 ton hard coal	= 34½ cu. ft.	1 meter	= 40 in.
1 ton soft coal	= 38 cu. ft.	1 decimeter	= 4 in.
1 cu. ft.	= .8 bu. level full	1 centimeter	= .4 in.
2747.07 cu. in.	= 1 heaped bushel	1 kilometer	= .6 mi.
4000 cu. in.	= 1 bu. of ear corn	1 liter	= 64 cu. in.
7½ gal.	= 1 cu. ft.	1 liter	= 1.1 liquid qt.
1 gal. water	= 8½ lb.	1 hektoliter	= 2.8 bu.
31½ gal.	= 1 barrel (bbl.)	1 gram	= .035 oz.
196 lb.	= 1 bbl. of flour	1 kilogram	= 2.2 lb.

Weights per Bushel

In estimating crop production and values, the United States Bureau of Crop Estimates regards the bushel as a definite weight instead of a definite volume. These are the bushel weights used for 16 crops:

Wheat, potatoes, clover and alfalfa seed, 60 lbs.	Corn in ear . . . 70 lb.
Shelled corn, rye, kaffir corn 56 lb.	Onions 57 lb.
Sweet potatoes, turnips 55 lb.	Timothy seed . 45 lb.
Barley, buckwheat, apples, peaches 48 lb.	Oats 32 lb.

Condensed Multiplication and Division Tables to 12's

1	2	3	4	5	6	7	8	9	10	11	12
2	4		To use as a multiplication table, read to the right to the								
3	6	9	end of the line and then down; thus, $3 \times 2 = 6$, $3 \times 3 = 9$								
4	8	12	16	end of line; then down $3 \times 4 = 12$, $3 \times 5 = 15$,							
5	10	15	20	25	etc. To use as a division table, read up						
6	12	18	24	30	36	to the top of the column; 4's in 48, 12;					
7	14	21	28	35	42	49	4's in 44, 11; and so on to 4's				
8	16	24	32	40	48	56	64	in 16; then to the left			
9	18	27	36	45	54	63	72	81	4's in 12, 3; 4's		
10	20	30	40	50	60	70	80	90	100	in 8, 2.	
11	22	33	44	55	66	77	88	99	110	121	
12	24	36	48	60	72	84	96	108	120	132	144
1's	2's	3's	4's	5's	6's	7's	8's	9's	10's	11's	12's

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ANSWERS—EIGHTH GRADE

CHAPTER I

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VI

171	194	350	298	374	183	215	225
-----	-----	-----	-----	-----	-----	-----	-----

VII

608	691	196	143	243	389	226
130	357	568	187	619	169	159

VIII

12,670	6,150	28,445	61,911	19,470	9,416	24,668
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IX

9,742	3,546½	5,696	8,340	7,921
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X

26,812	31,653	21,334	36,730	30,808	26,366	33,246
--------	--------	--------	--------	--------	--------	--------

XI

3,208,861	2,413,528	2,119,842	3,244,604	939,176	3,138,336
-----------	-----------	-----------	-----------	---------	-----------

XII

24,802	6,187	1,058	7,634
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XIII

1¾	1	1	2	1⅞	1⅝	1⅞	1⅔	1	27½	1¾	1⅓
----	---	---	---	----	----	----	----	---	-----	----	----

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XIV

5,137	4,916	4,452	4,954	4,225	6,945	4,965	4,803
4,922	3,330	5,038	5,019	4,992	4,253	5,034	4,120

XV

33,965,486	6,486,523	1,290,730	46,518,496
2,627,658	9,902,582	1,145,753	61,727,773
2,710,803	54,281,097	353,898	14,914,409

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XVI

190,727	232,503	651,360	411,978
440,370	128,041	359,233	552,392
162,840	628,144	319,976	724,672
216,144	207,566	3,147,120	101,232

XVII

12,576,442	50,810,979	16,617,783
17,110,820	79,838,560	60,508,760

XVIII

293	705	892	411	851
641	650	483	742	384
714	273	940	337, rem. 40	809

XIX

$1\frac{1}{8}$	$3\frac{1}{16}$	$1\frac{1}{4}$	$3\frac{1}{8}$	$3\frac{3}{4}$	$3\frac{3}{4}$	$8\frac{5}{8}$	$1\frac{3}{4}$
$1\frac{1}{2}$	$33\frac{1}{3}$	$37\frac{1}{2}\%$	$62\frac{1}{2}$	$12\frac{1}{2}$	$16\frac{2}{3}\%$	$37\frac{1}{2}\%$	$4\frac{1}{16}$
$12\frac{1}{16}$	$37\frac{1}{2}\%$	$16\frac{2}{3}\%$	$\frac{1}{6}$	$70\frac{5}{6}\%$	$\frac{1}{24}$	$10\frac{5}{12}\%$	$\frac{5}{8}$

Page 207.

XX

1. 47.	2. 45.	3. 33	4. 6.	5. 2.	6. 5.	7. 15.	8. 31.
9. 20.	10. 20.	11. 33.	12. 50.	13. $43\frac{1}{2}$.	14. 17.	15. 1.	16. 163.
17. 19.	18. 48.	19. 37.	20. 56.				

XXI

300%	31	$218\frac{1}{8}$	$126\frac{2}{3}$	2,200	290	$2\frac{1}{4}$	10	275
------	----	------------------	------------------	-------	-----	----------------	----	-----

XXII

845	916	436	278	951	273
-----	-----	-----	-----	-----	-----

Page 208.

XXIII

	a	b	c	d	e
1.	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{8}{10}$	$\frac{7}{12}$
2.	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{7}{9}$	$\frac{5}{9}$
3.	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{10}$
4.	$\frac{1}{6}$	$\frac{2}{8}$	$\frac{1}{2}$	$\frac{2}{9}$	$\frac{1}{6}$
5.	$1\frac{1}{2}$	$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{1}{5}$	$\frac{2}{3}$
6.	$2\frac{2}{3}$	3	$3\frac{1}{3}$	$1\frac{3}{5}$	3
7.	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{1}{4}$	$\frac{2}{5}$
8.	1	$1\frac{1}{2}$	$\frac{2}{3}$	$\frac{8}{9}$	$1\frac{1}{8}$
9.	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{2}{10}$	$\frac{5}{9}$	$\frac{3}{10}$
10.	$\frac{7}{10}$	$1\frac{1}{10}$	$3\frac{1}{2}$	$\frac{2}{9}$	$\frac{5}{16}$
11.	$\frac{1}{8}$	$3\frac{1}{5}$	$3\frac{1}{2}$	$\frac{7}{12}$	$\frac{1}{12}$

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	a	b	c	d	e
12.	$8\frac{4}{5}$	$3\frac{1}{8}$	$2\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{8}$
13.	$1\frac{1}{4}$	$2\frac{1}{7}$	$2\frac{1}{8}$	$3\frac{2}{9}$	$3\frac{1}{9}$
14.	$\frac{3}{4}$	$\frac{1}{7}$	$\frac{3}{16}$	$\frac{2}{25}$	$\frac{8}{28}$
15.	$\frac{1}{2}$	$1\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{9}$	$\frac{1}{6}$
16.	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{3}{5}$	$1\frac{1}{2}$	$\frac{7}{8}$
17.	$\frac{1}{2}$	$\frac{7}{24}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{3}$
18.	$2\frac{2}{5}$	$2\frac{2}{5}$	$1\frac{2}{7}$	$1\frac{2}{7}$	$1\frac{2}{7}$
19.	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{5}{16}$
20.	$\frac{1}{5}$	$1\frac{1}{4}$	$\frac{3}{5}$	$1\frac{1}{8}$	$1\frac{1}{5}$
21.	$1\frac{1}{2}$	$\frac{1}{8}$	$1\frac{1}{2}$	$2\frac{2}{3}$	2
22.	20	$1\frac{1}{16}$	$2\frac{1}{4}$	$\frac{3}{16}$	$4\frac{2}{3}$
23.	$4\frac{1}{2}$	$\frac{1}{12}$	$2\frac{1}{12}$	$5\frac{1}{4}$	$11\frac{1}{2}$

Page 210.

XXIV

- (1) .01 (2) 100 (3) 2. (4) .5 (5) 200 (6) 5. (7) .8 (8) 1.
 (9) 10 (10) .4 (11) 10. (12) 40 (13) .04 (14) 1. (15) 5,000 (16) 3,200.
 (17) 85. (18) .004 (19) 8.2 (20) .021.

XXV

	a	b	c
1.	1	.01	10
2.	100	3.	300
3.	.1	10.	.04
4.	.04	40.	9.
5.	10.	1,000.	1.
6.	2.	.5	.5
7.	.05	.004	.4

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	a	b	c		a	b	c
8.	.4	.1	10.	12.	50.	5.	.2
9.	.1	10.	10.	13.	80.	5.	.1
10.	400.	3,200.	5.	14.	100.	10.	1.
11.	.2	.25	5.	15.	100.	10.	.1

XXVI

	a	b	c
1.	1,900.	3,300.	\$450.
2.	49,600.	5,625.	56.25
3.	3,300.	\$375.	4,000.
4.	3.	800.	800.
5.	10.5	800.	1,235.
6.	2,499.	693.	.4
7.	3.	290.	70.7
8.	20.	\$1,320.	\$900.
9.	\$3,025.	\$9,025.	99.
10.	\$10.10	6,336.	2,379.
11.	\$1,080.	57.	6.25
12.	1,728.	1,225.	800.
13.	1,200.	15.	\$51.50
14.	\$5.	12 ft.	\$5.
15.	77.	2,250.	13,225.

Page 212.

XXVII

1.	6	12	50	$1\frac{1}{2}$
2.	25	100	2	60
3.	75	30	20	250
4.	18	32	60	9
5.	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{1\frac{1}{2}}$	$\frac{1}{2}$
6.	36	81	12	18
7.	$2\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{16}$
8.	27	24	144	30
9.	1	.4	1.6	4.8
10.	125	100	625	10
11.	4	21	16	56
12.	100	900	40	135

XXVIII

1. 3.8 2. 7.5 3. .714 4. 87.6% 5. 54.6¢ 6. 50.6 yd.
 7. 9 yr. $3\frac{2}{3}$ mo. 8. 5 ft. $4\frac{1}{3}$ in. 9. 17.4 lb. 10. \$130.80.

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XXIX

1. 12. 2. 64. 3. \$1.50. 4. 24. 5. 9. 6. 10. 7. 8.
 8. 20. 9. 24. 10. 12. 11. 25. 12. \$37.50. 13. \$36. 14. \$9. 15. \$20.
 16. 80.

XXX

1. 331,628. 2. 31,491. 3. 603,296. 4. 7,702,032.
 5. 1,493,073- $\frac{2}{3}$. 6. (1) \$1.13. (2) \$2.75. (3) \$13.50. (4) \$1.43.

CHAPTER II

Page 214. 1. 37,836 2. 37,099 3. 31,488 4. 33,701 5. 38,521
6. 30,463 7. 39,153.

Page 215. 1. 235 2. 239 3. 219 4. 221 5. 289 6. 272 7. 223
8. 200 9. 273 10. 211
1. 900 2. 1400 3. 1000 4. 1600 5. 1600 6. 1800
7. 1700 8. 1500 9. 1300.

Page 216. 1. 7077 2. 6982 3. 8157 4. 8103 5. 7473 6. 7059
Line Totals: \$4109.26; \$6031.31; \$2886.69; \$4172.00; \$1715.41.
Grand Total: \$18,914.67.

Page 217. Total gross profit, \$8525.30; total expense, \$5121.45.

Page 218.	a	b	c
1.	\$8	\$6	\$ 4.25
2.	18.50	12.25	10.50
3.	7.75	3	4.50
4.	125	28	213
5.	96	63	128
6.	4.67	9.33	3.17

Page 219.	a	b	c
1.	\$6.50	\$1.75	\$4.25
2.	20.75	2.25	20
3.	9	8	5.63
4.	11.25	3.84	7

2

a	b	c
347,984	420,849	184,664
467,584	280,784	70,308
86,100	226,416	491,184

3

a	b	c
1. 1224	672	288
2. 4095	3132	62.4
3. 1260	442	66.42
4. 2915	85.56	774.3
5. 2256	4624	1296

Page 220.

	a	b	c
1.	1548	672	1275
2.	1449	3496	2475
3.	1974	1691	5460
4.	8004	7857	9506
5.	3132	1456	3154

5

	a	b	c
1.	222,976	145,488	895,470
2.	137,536	37,467	76,184
3.	498,278	523,710	211,322
4.	16,403.2	5,659.2	50,941

Page 221.

6

	a	b	c
1.	34,440	47,502	216,688
2.	14,256	8,064	121,737
3.	434	1,386	5,874
4.	1,422.4	236.28	1,495.04

Page 222.

7

	a	b	c
1.	182	592	20
2.	50	12	68
3.	13	7	6
4.	232	126	474
5.	1,031	84	9
6.	42	55	13

8

1. 28%, \$604.80 2. $46\frac{2}{3}\%$, \$1066.67 3. 23.5%, \$688.50
 4. 14.5%, \$513 5. 32.5%, \$202.50 6. $66\frac{2}{3}\%$, \$133.33 7. 49%, \$20.40
 8. 68%, \$32 9. 27.75%, \$578 10. 19%, \$48.60 11. 9.75%, \$180.50.

Page 223.

1

	a	b	c
1.	1.68	9.68	1.408
2.	7.68	28.48	6.736
3.	5.04	50.4	2.184
4.	4.2	12.6	2.94
5.	2.52	20	2.208
6.	2.94	4.92	7.5
7.	29.4	65.6	31.28

	a	2 b	c
1.	498.1	48.2145	43.04+
2.	330.8	29.173	37.8—
3.	641.2	153.18+	29.19

Page 225. 1. 1.0 2. 1.2 3. 1.4 4. 1.3 5. 1.2 6. 1.1 7. 1.9
8. 2.9.

PROBLEMS

1. 38.4% 2. 11.7% 3. 50.1+% 4. 1.7—% 5. 104.5+%.

CHAPTER III

Page 229.

	a	b	5 c	d	e
1.	37	97	53	91	34
2.	39	35	85	95	47
3.	61	73	79	76	93
4.	17	67	26	31	81
5.	87	58	59	66	42
6.	78	75	29	19	45
7.	55	83	65	52	23

	a	b	6 c	d	e
1.	.55	.22	.85	.95	.97
2.	.65	.79	.75	.45	.58
3.	.67	.25	.52	.56	.35
4.	.41	.53	.78	.61	.73

	a	b	7 c	d
1.	6.5	5.5	2.5	9.6
2.	1.4	1.3	9.5	7.2
3.	3.3	5.7	2.3	9.3
4.	3.9	5.6	4.6	8.7

Page 231.

	a	b	3 c	d	e
1.	5.8+	95.5+	.94+	.83+	39.0+
2.	1.8+	85.6—	.47+	.63+	54.7+
3.	8.8+	11.2—	.19—	9.5—	16.9—
4.	8.4—	6.5+	.75+	6.0+	5.0+

III

	a	b	c	d	e
1.	$-\frac{3}{8}$	$1\frac{1}{3}$	$1.5+$	$.61+$	$2.5+$
2.	$\frac{2}{3}$	$5\frac{1}{2}$	$3.5+$	$.81+$	$2.7-$
3.	$1-\frac{3}{8}$	$1\frac{3}{4}$	$1.6+$	$1.6+$	$4.3+$

Page 232.

II

	a	b	c	d	e
1.	7.5	$.94+$	85	$9.8-$	$12.2+$
2.	9.5	$.36+$	91	$8.6+$	$11.4+$
3.	$2.9-$	$.18$	76	$5.9+$	$21.0+$

CHAPTER IV

Page 234. 4. (1) 29; 65. (2) 28; 39. (3) 77; 117.

Page 235. 5. 35 ft. 6. 40 in. 7. 27 ft. 8. $\frac{3}{4}$ in. 9. 70 ft. 10. 80 rd. 11. 40 ft. 12. 69 ft.

Page 237. 10. 95 ft.

Page 238. 11. (1) $127.27+$ ft. (2) $127.27+$ ft. (3) $227.27+$ ft. (4) $63.6+$ ft. (5) $228.14+$ ft. 12. $234.32+$ mi. 13. $4.9+$ mi. 14. 25 mi.Page 239. 8. $75.6-$ ft. 9. A 24 in.; 216 sq. in.; B 30 ft., 216 sq. ft.; C 32 rd., 1920 sq. rd.; D 125 yd., 2100 sq. yd.; E 5 ft., 6 sq. ft.; F 4 in., 6 sq. in.; G 143 yd., 3630 sq. yd.; H 17 ft., 60 sq. ft.; I 5 in., 30 sq. in. 10. (1) 15 ft.; $16\frac{1}{2}$ ft.; 1617 sq. ft. 11. 2522 bd. ft.; \$100.88.

CHAPTER V

In this chapter answers vary in accordance with the degree of accuracy of measurement. The answers here given are based on measurement to sixteenths and in certain problems one half of one sixteenth of an inch.

Page 243. 2. 55.9 ft. 3. $\frac{3}{4}$ 4. 25 rd. 5. Use scale measurement. 6. Subtract the distance A F, found by actual measurement, from A B. 7. $\frac{2}{3}$; $\frac{2}{3}$

Page 244. 10. $\frac{1}{2}$; $\frac{1}{2}$; 50 rd. 11. By ratio; by scale measurement.

Page 245. 12. 26 ft. 8 in. 13. 75 ft. 2. (1) 12 ft. (2) $13-\frac{5}{8}$ ft. (3) $1-\frac{3}{8}$ ft. (4) 9 ft.

Page 246. 3. $19-\frac{3}{8}$ ft.; $5-\frac{1}{8}$ ft.; $10-\frac{3}{8}$ ft. 4. 4 ft. 5. 10 ft. 1. 975 ft.; 6075 ft

Page 247. 2. 15.13—acres. 4. 100 ft.; 50 ft. 5. 350 ft.; 500 ft.; 550 ft. 6. \$45,937.50 7. 30,833 $\frac{1}{3}$ 9. Jones has 80% more than Smith. 10. Miss Ross has 2.2 times as much land as there in in block b.

Page 248. 1. 30.6—rd. 2. 26,400 ft. 3. 8.6+ ft.; 43+ sq. ft. 4. 45+ sq. in. 5. Yes 6. 95 ft. 7. 16; 16 ft. 8 in.

CHAPTER VI

Page 250. 10. Almost \$810, counting 2,150.42 cu. in. to the bushel. 11. 20.869+tons; 521.7+bu. 13. 1,000 cu. ft. 14. 200 cu. ft. 15. 3,216 cu. ft. 16. 1,416.3—bu., counting 1 cu. ft.=.8 bu.

Page 252. 8. $\frac{63}{8} \pi$ cu. in. 9. 80 π cu. in. 10. 28.04—cu. in. 11. 9.18 gal. 7. 10 π sq. ft.

Page 253. 8. \$7.54 9. 35.95+sq. ft. 1. 13.571 2. \$10.86 3. 13.5+sq. ft. 4. 41.56+sq. ft. 5. \$27.54 6. 9.5+in. 7. 424.1+gal. 8. 2.45 gal.

Page 254. 1. 6 ft. x 6 $\frac{1}{2}$ ft. 2. 7.347+ T.; \$91.84 3. 1.14+ T. 4. 1354.8+ T. 5. 1561.65+cu. yd. 6. 14.7+ lb. 7. 54 π cu. ft.

Page 255. 2. \$277.20 3. 124 $\frac{1}{8}$.

Page 256. 4. 28 bd. ft. 6. \$108.

CHAPTER VII

Page 261. 1. 18 2. 18 3. 16 4. 35 5. 99 6. 17 7. 100 8. 5 9. 4 10. 174 11. 7 12. 32 13. 5 14. 100.

Page 262. 15. 21 16. 2 17. 21 18. 20 19. 6 20. 3 21. \$6 22. \$7 23. 2 24. 30 25. 400 26. 17 27. 40 28. 64 29. 70 30. 22 31. 25 32. 12 33. 80 34. 200 35. $\frac{1}{2}$ 36. 2 37. 4 38. 2 39. 18 40. 32 41. 1 42. 2 43. 2 44. 3 45. 6 46. 36 47. 256 48. 2401 49. 10 50. 7 51. 12 52. $1\frac{5}{8}$ 53. 7 54. 6 55. 6 56. 4 57. 96 58. 7 59. 10 60. 5 61. 12 62. 18 63. 5.

Page 263. 1. 30 2. \$4 3. 25 $\frac{5}{8}$ gal.

Page 264. 4. 29 5. 28 6. 28 7. \$1.40 8. \$27 9. 10 10. 8 11. 24¢ 12. 172 13. 66.

Page 265. 14. 6 $\frac{1}{2}$ 15. 10 in. 16. 300 17. 4000 18. 1500 19. 10 20. \$4 21. 60 22. \$700 1. 2¢; 6¢ 2. 6; 24.

Page 266. 3. 5; 10 4. 12; 8 5. 64; 128 6. 16 in.; 8 in. 7. 135;
142 8. 153 bu.; 306 bu.; 1224 bu. 9. \$6; \$12; \$3 10. 16 oz.; 32 oz.; 48 oz.

Page 267. 1. 90° 2. 40° 3. 45° ; 45° ; 90° 4. 30° ; 60° ; 90°
5. 30° ; 60° 6. $22\frac{1}{2}^\circ$; $67\frac{1}{2}^\circ$; 90° 7. 90° ; 18° ; 72° 8. 100° ; 20° ; 60°
9. 90° ; $67\frac{1}{2}^\circ$; $22\frac{1}{2}^\circ$.

Page 272. 1. $P=56$; $a=16$; $b=10$ 2. $A=100$; $b=16$ 3. $A=27$; $a=12$
4. $A=252$; $a=8\frac{2}{3}$; $b=8$; $b'=10$ 5. $A=96$; $d=9$; $d'=8$.

Page 273. 6. $h=13$; $a=8$; $b=10$ 7. $h=41$; $a=7$; $b=60$ 8. s.h.=20;
 $a=36$; $R=8$ 9. $V=320\pi$; $a=12$; $R=6$.

Page 274. 3. 160 rd. 4. 192 sq. ft. 5. 16 ft. 6. 144 sq. ft. 7. 60 sq.
in.; $34.3 + \text{in.}$ 8. 6,283,200 sq. mi. 9. 201,062,400 sq. mi.

Page 278. 1. 537.6 bu. 2. $21.7 + \text{tons}$ 3. $2,025 \pi$ cu. ft. 4. 324π cu.
ft. 5. 4.81 cu. in. too low 7. Yes 8. 37 9. $4,500 \pi$ gal.

Page 279. 10. 69.96π Sq. ft. 11. 50 sq. in. 12. (1) $45.3 + \text{ft.}$, $102.6 + \text{ft.}$;
(2) 180 sq. ft.; (3) 15% 13. $10.39 + \text{ft.}$, $62.34 + \text{sq. ft.}$ 14. 12; 108
15. 10 ft.

Page 280. 16. 1150π tons 17. $1175 + \text{gal.}$ 18. \$19.70 19. 12 ft.
20. (a) 6 in.; $\frac{3}{4}$ in.; $\frac{1}{2}$ in. (b) $4\frac{1}{2}$ in. x 4 in. (c) $68 + \%$ (e) $5\frac{3}{4}$ sq. in.; 25 in.

Page 281. 1. $21.2 + \text{tons}$ 2. $172 - \text{days.}$ 3. 20 4. 30.

Page 282. Areas of small figures: $1\frac{1}{2}$ sq. in., $\frac{5}{16}$ sq. in., $\frac{1}{4}$ sq. in., $\frac{5}{8}$ sq.
in., $1\frac{1}{8}$ sq. in., $\frac{3}{8}$ sq. in., $1\frac{1}{2}$ sq. in., $\frac{9}{16}$ sq. in., $\frac{9}{16}$ sq. in. Total area of small
figures: $6\frac{13}{16}$ sq. in. $34\frac{1}{16}\%$.

Page 283. 1. $32.73 + \text{acres}$ 2. 25.5 acres 3. Smith should pay Brown
\$41.32.

Page 285. 1. 17,176.7 - gal. 2. 4 hr. 37 - min. 3. 60 - gal.
4. 0.000737¢; 1,356 + 5. $139 + \text{gal.}$; 38,069.5 millions 6. $37.4 + \text{¢}$
7. $13,757 + \text{tons}$ $1,748 + \text{tons}$; $1,491 + \text{tons}$

CHAPTER VIII

Page 292.

The answers in this exercise (pages 292 and 293), are approximations which
are sufficiently accurate for reading purposes.

1. $83\frac{1}{3}$ 2. $333\frac{1}{3}$.

Page 293. 3. Metz, $21\frac{7}{8}$; Strasburg, $106\frac{1}{4}$ 4. $333\frac{1}{3}$ 5. $13\frac{3}{4}$
6. $3\frac{1}{3}$ 7. $83\frac{1}{3}$ 8. $1666\frac{2}{3}$ 9. 3 ft. x 5 ft.; 4 ft. 10. 10
11. 3-inch and $3\frac{1}{2}$ -inch guns.

CHAPTER IX

Page 296. 1. \$250, 10%. 2. 25.9+%. 3. 8.1+%.

Page 297. 5. Apples, \$1.50 more profit; walnuts, 15.7- % greater rate of profit. 6. $18\frac{2}{3}\%$, $25\frac{1}{3}\%$. 7. \$1.85, 154+%. 8. \$3.865+. 9. \$1,645.80.

Page 299. 1. \$20.83.

Page 301. 1. .26-cu. yd.; 51+cu. yd.; .77+ cu. yd.; .22+cu. yd.; .44+ cu. yd.; .88+cu. yd.; .15- cu. yd.; .44+cu. yd.; .88+cu. yd. 2. 21 bbl., 7 cu. yd., 10 cu. yd. 3. 38 bbl., 6 cu. yd. 4. 117 sacks, 9 cu. yd., 13 cu. yd. 5. 45 sacks, 3 cu. yd., 3 cu. yd. 6. \$1,050, \$255, \$332.64. 7. \$1,281, \$507.60, \$1,015.20.

Page 302. 1. \$67,193+. 2. 79. 3. 34.7- %. 4. \$.665.

Page 303. 5. \$176.40 6. \$2.70. 7. \$66. 8. \$27.76. 9. \$.937+. 10. \$2.487+. 11. Yes. 12. 51.7+¢ per bu. better at Liverpool.

Page 304. 14. Full car lot rate is \$40.96 better. 15. \$6,820. 1. 3,-000,000. 2. \$3,750,000. 3. 5,000,000.

Page 305. 4. 25,097.

Page 307. 1. \$5.87.

Page 308. 2. \$3.87. 3. \$7.14.

Page 309. 1. 37¢. 2. 46¢. 3. 43¢. 4. 37¢, 9¢, 26¢, 86¢, 17¢, 19¢, 56¢, 19¢, 35¢, 29¢. 1. Express is 2¢ cheaper. 2. Express is 10¢ cheaper.

CHAPTER X

Page 317. 2. \$138.36. 3. No.

Page 318. 4. \$103.52. 5. \$1.

Page 319. 1. (1) \$115.93; \$119.41. (2) \$19.41. (3) \$1.41.

Page 322. 1. \$134.39. 2. \$4.12 compounded at 4% for 5 yr. amounts to \$5.01. 3. Exactly 4%. 4. Yes. 5. \$4.22, compounded at 4% for 4 yr. 2 mo. amounts to \$4.97. 6. Compound int. is \$3.87 more. 7. \$1,689.93.

CHAPTER XI

Page 323. 1. \$211.69. 2. \$123.58. 3. \$4,127.27.

Page 324. 3. Fish, 131+%; tomatoes, 15+%, chicken, 50%; cabbage, 18+%; oranges, 40%; English walnuts, 126+%; sirloin, 126+%; bananas, $33\frac{1}{3}\%$.

Page 325. 4. 24¢ ; $23\frac{1}{3}\text{¢}$; 15¢ ; 11¢ ; $27.6+\text{¢}$. 8. $47.4-\%$. 9. 200% .
11. 20% . 12. 100% .

Page 326. 1. $36.3+\%$. 2. $65.2+\%$. 3. $\$3.37+$. 4. (1) 380.1735 tons.
(2) $\$1,960.76-$. (3) $\$479.86$. (4) $\$9,872.47$. (5) $\$12,203.09$. (6) $\$21,965.56$.

Page 327. 1. $\$226.50$. 2. $\$4.53$. 3. $\$93.30$. 4. $\$1.87-$. 5. $\$133.20$.
6. $\$2,664$. 7. $62+\%$. 8. $142.7+\%$.

Page 328. Total cost, $\$22.45$; value of corn, $\$81.60$; net profit, $\$59.15$.

Page 329. II. (1) $\$585$. (2) Taxes. (3) $\$31$. III. $33-\%$.

Page 330. IV. (1) 3,640 on a 26 bu. basis. (2) $7.8-\text{oz}$. (3) $10\frac{2}{3}\text{ oz}$.
V. $\$.0705$.

Page 331. I. (1) $1.63+\text{lb}$.; $1.77+\text{lb}$.; $2.92+\text{lb}$.; $2.79+\text{lb}$. II. (1) First.
(2) Third. (3) Second.

Page 332. 5. $\$1.70$ profit, not counting labor cost. III. (1) The value
of $3\frac{1}{2}\text{ lb}$. of shelled corn. (2) The value of 12 lb. of gain in wt. IV. (1) 8
lb. (2) 2 lb. shorts, $\frac{2}{3}\text{ lb}$. oilmeal.

Page 333. VI. (1) Gain 49.2¢ on each bu. fed. (2) $\$393.60$ gain. VII. (1)
 $\$3.32$. (2) 40. (3) 80; yes, because he could have produced two times as
much gain.

Page 334. I. (1) 48. (2) 175. (4) $102-\%$ in 1914; 70% in 1915; $97+\%$
in 1916; $35+\%$ in 1917; $61-\%$ in 1918.

Page 335. II. (1) Jan., 51; Feb., 12; Mar. 455; Apr., 443; May, 427;
June, 437; July, 539; Aug., 520; Sept., 485; Oct., 294; Nov., 12; Dec., 22.
(2) 3,697.

Page 336. (4) Jan., $\$2.55$; Feb., 55¢ ; Mar., $\$15.17$; Apr., $\$12.55$; May,
 $\$10.68$; June, $\$10.56$; July, $\$12.13$; Aug., $\$12.57$; Sept., $\$12.53$; Oct., $\$8.58$;
Nov., 50¢ ; Dec., $\$1.14$. (6) $\$79.51$. III. (1) Balance, $\$45.15$.

Page 337. IV. (1) 8 ft. $7\frac{1}{2}\text{ in}$. (2) 468 bd. ft., counting the roof 16 ft.
 $\times 8\text{ ft. }7\frac{1}{2}\text{ in}$. (4) $\$8.19$. 1. 19. 2. $\$700$. 3. 60¢ , 90¢ , $\$1.50$. 4. $34+\%$.
5. Lacked $1\frac{1}{3}\text{ oz}$. on each piece of being accurate.

Page 338. 6. 10¢ . 7. $\$132$. 8. 21¢ . 9. $5.3-\%$. 10. 45,000,000.

CHAPTER XII

Page 342. 1. $\$62.67$. 3. (1) $\$7.50$; $\$32.50$. (2) 9 mo after buying the
note. (3) No. Miller's investment is $\$532.50$. (4) Endorsed the note.

Page 345. 1. $\$100,000$. 2. 10,000. 3. $1\frac{2}{3}\%$. 4. $\$83.25$. 5. $\$6$; $\$3$; 6¢ .

Page 347. 4. 2,300 shares. 5. $4.4+$. 6. 193 shares; $\$25.42\text{ rem}$.

Page 348. 7. \$98,100. 8. \$10,830. 9. \$4,335. 11. \$382.50. 12. \$1,070.

Page 349. 1. Less, because bought at a prem. 2. No; 80. 3. $8.69 + \%$.
4. $6\frac{2}{3}\%$. 5. \$200 for each \$100.

Page 352. 8. 86¢. 9. \$268.08. 10. \$9,812.50.

Page 353. 12. 54; \$63.10, rem. 13. One; \$16.92, rem.

Page 355. 10. $5.37 + \%$. 11. $4.66 + \%$. 12. $2.99 + \%$. 13. \$84.40 compounded at $4\frac{1}{4}\%$ for 4 yr. 1 mo. amounts to \$100.04. 14. \$824 compounded at 4% for 5 yr. amounts to \$1,002.56.

Page 357. 1. $9.87\frac{1}{2}\%$. 2. \$21.71. 4. \$78.90; \$347.35.

Page 358. 1. First is \$7 better if the 6% investment is tax free.
2. \$582.97 more. 3. \$31.37. 4. Loss is \$16.60 yearly by owning the house.
5. 1st, \$75; 2nd, \$68.75 yearly.

Page 359. 1. 2% ; $33\frac{1}{3}\%$. 3. (1) $8.69 + \%$. (2) \$1,150. (3) \$253.
(4) $23\frac{1}{3}\%$ on selling price, 30.43% on cost. (5) $6\frac{2}{3}\%$. 4. \$1,455.44 if savings account interest is used.

CHAPTER XIII

Page 361.

I

6. 57434.

II

Mon. \$2165. Tues. \$1805.22. Total. \$12839.61.

Cashier's Pay. \$128.39.

Page 362. The grade should be $93\frac{2}{3}\%$. 5. c. 88. 6. d. 17781.

Page 363. I. 2. a. $38\frac{2}{3}\%$; c. 5. 4. b. $24\frac{5}{6}\%$. II. (5) 469821. III. (5) 939176. V. (2) 19800. VI. (5) .245.

Page 364. I. 1. c. $1\frac{1}{2}$. 4. b. $\frac{1}{2}$; c. $\frac{2}{3}$. II. 1. b. 0.01; d. 600. 3. b. 8.
4. c. 1. III. 5. a. 25; b. 8; c. 2.1.

Page 365. First Acct. Balance \$56.20, Total \$130.40. Second Acct. Labor, 5 da. @ \$1.90, \$9.50; Labor $5\frac{1}{2}$ da. @ \$2.00, \$11; Balance \$12.10.
Third Acct. Balance, \$15.60.

Addition

Page 366. First row. 5th column from left, 4954. Second row. 3rd column, 4922; 6th column, 5019.

Division

First row. 1st example, 572. Second row, 1st example, 925; 3rd example, 534.

Subtraction

Page 367. First row. 2nd example, 30970095; 3rd example, 8238767; 4th example, 81919839.

Multiplication

First row. 2nd example, 190727. Second row. 1st example, 440370.

Page 368.

I

	a	b	c
1.	\$48.	\$1.20	80¢
2.	63	6	30¢
3.	81	6	48¢
4.	18	3.60	25¢
5.	28	2.85	64¢

II

	a	b		a	b
1.	\$28.60	\$120	7.	25.2	300
2.	76.50	225	8.	18.81	441
3.	68.40	262.5	9.	57	114
4.	18.75	213.75	10.	6.75	615.6
5.	13.5	547.2	11.	13.5	384.75
6.	39.2	218.7			

Page 369. 6007

4437

4497

5792

Page 370.

Set A

2,710,803	54,281,097	353,898	14,914,409
33,965,486	6,486,523	1,290,730	46,518,496
2,627,658	9,902,582	1,145,753	61,727,773

Set B

1	2	3	4	5	6
48,634	64,965	53,433	12,906	30,465	18,496
7	8	9	10	11	12
7,653	19,926	2,572	45,753	58,977	10,678
13	14	15	16	17	18
37,118	2,927	14,261	2,888	2,409	83,195

Page 371.

Multiplication

322,418	321,604	214,542	243,340
142,830	393,255	647,924	474,776

Division

845	916	436
278	951	273

Common Fractions

	a	b	c	d
1.	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{7}{9}$
2.	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{12}$
3.	$1\frac{1}{2}$	$\frac{2}{3}$	3	10
4.	$\frac{1}{3}$	$\frac{5}{9}$	$\frac{2}{6}$	$\frac{1}{12}$
5.	10	10	$11\frac{1}{2}$	$19\frac{1}{3}$
6.	1	2	$\frac{1}{2}$	$2\frac{1}{4}$
7.	$\frac{1}{32}$	$\frac{1}{8}$	$\frac{5}{24}$	$\frac{1}{12}$
8.	$1\frac{1}{4}$	$2\frac{1}{4}$	$3\frac{1}{8}$	$6\frac{1}{5}$
9.	$3\frac{1}{9}$	$3\frac{7}{8}$	$1\frac{19}{32}$	$1\frac{3}{4}$
10.	$6\frac{1}{4}$	$8\frac{1}{4}$	$15\frac{1}{8}$	$30\frac{5}{16}$

Page 372.

Set A

1. 22.10	2. .5100	3. 3.84	4. .384
5. .00625	6. .352	7. .00056	8. 40.96
9. 1562.5	10. 18.225	11. 58.75	12. 1.156
13. 71.442	14. .3105	15. 15.00	16. 12.016
17. 125.6640	18. .9750	19. .00001	20. .0100

Set B

1. 27.5	4. 2.75	7. 27.5
2. 27.5	5. .0275	8. 275
3. 27.5	6. 27500	9. 2750

Page 373.

Set C

a	b	c
1. 22.10	.5100	3.84
2. .384	.00625	.032
3. .00056	40.96	1562.5
4. 18.225	5625.	42.25
5. 4.2250	58.75	1.156
6. 71.442	.3105	15.
7. 12.016	125.6640	.9750
8. .00001	.0100	4.

Cancellation Test

1. 108; $16\frac{2}{3}$. 2. $1\frac{1}{6}$; $11\frac{2}{3}$. 3. $5\frac{5}{8}$; 8. 4. 108; 4. 5. $\frac{1}{2}$; 108.

Page 374.

I

4120	5299	4649	4975	5005	4912	3330
5038	5019	4954	4225	6945	4965	4803

II

1. 69. 2. 95. 3. 79. 4. 86.

Page 375.

III

a	b	c	d	e	f
1. 13149	30525	31727	31847	32948	33949
2. 36753	24289	32571	22496	25294	25948
3. 26812	31653	21334	36730	30808	26366

IV

\$1764.76	\$777.34	\$1210.45	\$950.25	\$1552.56
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Page 376. 1. 3 2. 3.2% 3. 30 4. 24 5. 28 6. 40 7. 35 8. 45
 9. 55 10. .52 11. 65 12. 36 13. 36 14. 36 15. $\frac{2}{3}$ 16. 75 17. .375
 18. $\frac{3}{8}$ 19. $83\frac{1}{3}$ 20. .35 21. 600 22. 300 23. 75 24. 800 25. 72 26.
 100 27. 90 28. 60¢ 29. \$4.50 30. 4 31. 15000 32. $\frac{3}{5}$ ¢ 33. \$500
 34. \$1200 35. \$1250 36. 1500 37. 900 38. 2000 39. 1500 40. 700
 41. \$1000 42. \$6000 43. \$11.25 44. \$26 45. 500 46. \$64.80 47. 50¢
 48. .7425 49. 0 50. .1%.

Page 377. 1. 3. 2. 25. 3. $66\frac{2}{3}$. 4. \$1.84. 5. \$4. 6. \$64. 7. 80.
 8. $37\frac{1}{2}$. 9. 1.8 ft. 10. 50. 11. 4. 12. 9. 13. \$50. 14. 12 yd. 15. 200.
 16. 100. 17. 50. 18. $33\frac{1}{3}$.

The Test of Life

1. 331628. 2. 31491. 3. 603296. 4. 7702032. 5. $1493073\frac{2}{3}$. 6. (1)
 \$1.13; (2) \$2.75; (3) \$13.50; (4) \$1.43.

Page 388. 1. The daily loss in 1922 was $2\frac{1}{3}$ times that in 1915. 2. The
 per capita loss in the United States was 4.23+times that of Great Britain.
 3. The total loss of the first six years is about 70% of that of the last six years.
 4. About \$3,695 million.

Page 389. 5. Over 60%.

Page 390. 2. The first, 90¢. 3. $1\frac{1}{4}$ ¢ per bu. is \$1.56 better. 4. \$4000.
 5. \$10000. 6. 10%; \$4235. 7. $8.2+$ %.

Page 391. 3. Lot No. 1, 1040 bd. ft.; lot 2, 504 bd. ft.; lot 3, 720 bd. ft.
 lot 4, 363 bd. ft.; lot 5, 594 bd. ft.; lot 6, 520 bd. ft.; lot 7, 1248 bd. ft.; lot 8,
 1920 bd. ft.; lot 9, 3264 bd. ft.; lot 10, 624 bd. ft.; lot 11, 936 bd. ft.

4. \$79.84.

5. 2161 bd. ft.

6. \$158.88.

7. 11,733 bd. ft.

Page 393. 1. 48,700 lb. 2. 400,000 sq. mi. 3. 735. 4. nitre, 270 lb. charcoal, 54 lb.; sulphur, 36 lb. 5. \$90,000. 6. 51.25 tons if profit is computed on cost; 44.56+tons if profit is computed on the selling price. 7. \$175. 8. 178%.

Page 394. 1. 168. 2. 239. 3. 1084. 4. 532. 5. 2,200,336. 6. 356940. 7. Nearly 15 to 1. 8. More than $6\frac{1}{3}$ to one.

Page 395. 7. 10; 4. 17. \$30. 18. \$464. 19. 7200 uc. ft.

Page 396. 24. \$500. 25. 3 ft. 26. \$34. 27. 100 ft. 28. \$80. 29. \$500. 30. \$1960. 31. \$3060. 32. \$1280. 33. \$10800. 34. 2.

Page 397. 41. \$6. 42. \$188. 43. \$80. 44. \$750. 45. \$60. 46. 8%. 47. \$920. 48. \$50. 49. $5.4+\%$.

Page 398. 50. 8%. 51. $133\frac{1}{3}$. 52. 75. 53. 70 sq. in. 54. 40 sq. in. 55. $219.91+\text{sq. in.}$ 56. 549.78 cu. in. 57. $47.12+\text{sq. in.}$ 58. 1256.64 sq. in. 59. 418.88 cu. in. 64. \$588.24. 65. \$432. 66. 20%.

